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# UNIT ORGANIZATION OF THE TOPICS SIMILAR TRIANGLES AND LOGARITHMS IN COLLEGE-PREPARATORY MATHEMATICS

Submitted by

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(B.S. in Ed., Boston University, 1927)

In Partial Fulfillment

of the Requirements for the Degree

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# PART I

## UNIT ORGANIZATION OF THE TOPIC

## SIMILAR TRIANGLES

# CHAPTER I

# THE PROBLEM AND ITS BACKGROUND

## Statement of the Problem

The problem.-- The problem of this paper is to apply the basic principles and procedures set forth in Fundamentals of Secondary-School Teaching[1] and the course in the Unit Method in the Secondary School[2] to the unit organization and classroom presentation of two topics in college-preparatory mathematics in a general high school. The topics, similar triangles and logarithms, were prepared for tenth and eleventh grades, respectively.

## The Pupils to be Taught

Class groups.-- The first unit was prepared for two groups of college-preparatory pupils enrolled in the tenth-grade course in plane geometry, which, at that time, was elective for girls and required for boys in the college-preparatory curriculum. Group I was composed of twenty-six pupils, fifteen boys and eleven girls, all in the tenth grade. Group II contained twenty-seven pupils,

1/Roy O. Billett, Fundamentals of Secondary School Teaching, Houghton Mifflin Company, Boston, 1940, pp. xvi, 671.

2/"Unit Method in the Secondary School", Roy O. Billett, Instructor, School of Education, Boston University Second Semester, 1946.

fifteen boys and twelve girls, of whom three were in the eleventh grade and the remainder in the tenth. The classes met five periods a week for forty-one minute periods. Every Monday or Friday, at the discretion of the teachers, twenty minutes were allotted to giving and correcting an arithmetic test for all mathematics classes in the school under the direction of the head of the mathematics department. Group I met during the second period in the day at 9:26, the first period of work, which followed the home room period. Group II met at 12:55, following the close of the lunch period.

Statistics regarding the pupils.-- From the six-year-record card of each pupil it was possible to obtain the following information: chronological age, intelligence quotient based on the Otis Quick Scoring Test, previous yearly averages in letter scores for mathematics in grades seven, eight, and nine; also, from results of the Stanford Achievement Test, Form H, given for grade 9.1, grade placements were to be found in reading vocabulary, reading comprehension, arithmetic reasoning, arithmetic fundamentals, language, and spelling. This data is recorded in Tables 1 to 3, 15 and 16 in Appendix A.

Chronological ages.-- At the time the pupils began the unit, the chronological ages varied from fourteen years six months to sixteen years five months in Group I

and from fifteen to seventeen years in Group II. (Table 1). The median ages were fifteen years six months and fifteen years eight months for Groups I and II respectively.

Table 1. Chronological Ages of Pupils in Group I and Group II on May 1, 1946, When Starting the Unit on Similar Triangles.

| Ages in Years and Months | Group I Frequency | Group II Frequency |
|---|---|---|
| (1) | (2) | (3) |
| 16-6 to 17-0 | 0 | 2 |
| 16-0 to 16-5 | 3 | 6 |
| 15-6 to 15-11 | 11 | 12 |
| 15-0 to 15-5 | 11 | 7 |
| 14-6 to 14-11 | 1 | 0 |
| Totals | 26 | 27 |
| High | 16 years 5 months | 17 years 0 months |
| Low | 14 years 6 months | 15 years 0 months |
| Range | 1 year 11 months | 2 years 0 months |
| Median | 15 years 6 months | 15 years 8 months |

Intelligence quotients.-- The intelligence quotients for Groups I and II ranged from 110 to 165 and from 97 to 146 with medians of 122 for each group. (Tables 15 and 16). In Group II, the intelligence quotients of the lowest six pupils indicated that they might expect to have trouble with the abstractions of geometry.

Previous marks in mathematics.-- Previous marks in mathematics for the pupils in Group I and II were studied to see if they could explain difficulties which might occur in the study of the unit. (Tables 15 and 16, Appendix A). The arithmetic and algebra marks for the previous three years were given in letter grades, with the following interpretations: A - 90% or higher; B - 80% to 89%; C - 70% to 79%; L or Low - 65% to 69%; D - below 65%. L was the lowest passing mark and indicated probable failure if a sequential subject were elected the following year. D indicated failure. Assuming that the health and effort of the pupils were good and the marking system fairly reliable, the ninth grade algebra marks indicated that there were six pupils in Group I and three in Group II who might be considered "mathematically inept"[1] and who might be expected to have trouble with the unit. As events disclosed, these pupils had great difficulty with the items involving right triangles and with proofs of theorems.

Achievement tests.-- The Stanford Achievement Tests were given when the pupils were beginning ninth grade work.

1/ Report of the Harvard Committee, General Education in a Free Society, Harvard University Press, 1945, P. 163.

Table 2. Achievement Test Grades for Group I of the Unit on Similar Triangles as Derived from the Stanford Achievement Tests when Pupils were in Grade 9-1.

| Grade Levels | Reading Vocabulary | Reading Comprehension | Arithmetic Reasoning | Arithmetic Fundamentals | Language | Spelling |
|---|---|---|---|---|---|---|
| | Frequencies | | | | | |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) |
| 13.0 to 13+ | 3 | 6 | 7 | 7 | 6 | 6 |
| 12.0 to 12.9 | 7 | 4 | 3 | 2 | 1 | 3 |
| 11.0 to 11.9 | 12 | 8 | 8 | 11 | 8 | 7 |
| 10.0 to 10.9 | 1 | 4 | 2 | 2 | 4 | 3 |
| 9.0 to 9.9 | 1 | 2 | 5 | 2 | 3 | 2 |
| 8.0 to 88.9 | 1 | 1 | 0 | 1 | 2 | 4 |
| 7.0 to 7.9 | 0 | 0 | 0 | 0 | 1 | 0 |
| Total | 25 | | | | | |
| High Score | 13+ | 13+ | 13+ | 13+ | 13+ | 13+ |
| Low Score | 8.0 | 8.0 | 9.0 | 8.0 | 7.0 | 8.0 |
| Range | 5+ | 5+ | 4+ | 5+ | 6+ | 5+ |
| Median | 11.3 | 11.2 | 11.2 | 11.2 | 10.8 | 11.0 |

Table 3. Achievement Test Grades for Group II of the Unit on Similar Triangles as Derived from the Stanford Achievement Test when Pupils were in Grade 9.1.

| Grade Levels | Reading Vocabulary | Reading Comprehension | Arithmetic Reasoning | Arithmetic Fundamentals | Language | Spelling |
|---|---|---|---|---|---|---|
| | Frequencies | | | | | |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) |
| 13.0 to 13+ | 2 | 4 | 1 | 4 | 3 | 3 |
| 12.0 to 12.9 | 0 | 0 | 2 | 1 | 1 | 2 |
| 11.0 to 11.9 | 12 | 11 | 6 | 8 | 7 | 7 |

Table 3. (Concluded)

| Grade Levels | Reading Vocabulary | Reading Comprehension | Arithmetic Reasoning | Arithmetic Fundamentals | Language | Spelling |
|---|---|---|---|---|---|---|
| | Frequencies | | | | | |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) |
| 10.0 to 10.9 | 1 | 3 | 4 | 3 | 4 | 3 |
| 9.0 to 9.9 | 5 | 1 | 6 | 3 | 3 | 3 |
| 8.0 to 8.9 | 2 | 3 | 1 | 2 | 3 | 2 |
| 7.0 to 7.9 | 0 | 0 | 2 | 1 | 1 | 2 |
| Total | 22 (Five not available) | | | | | |
| High Score | 13+ | 13+ | 13+ | 13+ | 13+ | 13+ |
| Low Score | 8.0 | 8.0 | 7.0 | 7.0 | 7.0 | 7.0 |
| Range | 5+ | 5+ | 6+ | 6+ | 6+ | 6+ |
| Median | 11.3 | 11.4 | 10.5 | 11.5 | 11.0 | 11.1 |

The tables show ranges of from four to six years in reading vocabulary, reading comprehension, arithmetic reasoning, arithmetic fundamentals, language, and spelling. The medians in the above for Group I were 11.3, 11.2, 11.2, 11.2, 10.8, and 11.0, respectively. For Group II they were 11.3, 11.4, 10.5, 11.5, 11.0, and 11.1. The low scores for some pupils in Group I in language and for some in Group II in arithmetic reasoning, arithmetic fundamentals, language, and reading comprehension disclosed conditions which had to be considered in planning the core assignments and in choosing reference books. Basic Mathematics[1] and

1/William Betz, Basic Mathematics, Ginn and Company, Boston, 1943, ix and 502.

Today's Geometry[1] were chosen with these pupils in mind. The other books given in the pupils' reference list were well fitted for the abler pupils of the group.

Home background of the pupils.-- Almost without exception the pupils had fine home backgrounds. There were more single homes than apartments in the school district, and in many sections neighborhood associations had been formed for recreational and improvement purposes. Parents were active in parent-teacher associations in the five elementary schools in the district and kept up their interest in the education of their children throughout the junior and senior high schools. They attended parents' night four times a year in great numbers and visited school at other times by invitation or by appointment.

The parents of the pupils for whom the units were planned were, for the most part, educated, cultured people who were able to send their children to college. Many of them were college graduates. Of the fathers, there was a physician, bank teller, draftsman, telegrapher, shoe designer, bookkeeper, salesman, store manager, vice-president of an oil-equipment company, certified public accountant, and a teletype operator. There was a drug store owner, skilled carpenter, atore clerk, construction engineer, policeman, loftsman, radio commentator, dental technician,

1/David Reichgott and Lee R. Spiller, Today's Geometry, Revised Edition, Prentice-Hall, Incorporated, New York, 1944, xvi and 400.

laundry owner, insurance agent, Federal income tax collector, engraver, shipyard inspector, postman, machinist-foreman, and head estimator at a gear works. Others worked at skilled trades in the manufacturing plants in the district. The father of the pupil with the highest intelligence quotient was a milkman before he enlisted in the armed forces.

Extra-curricular and out-of-school activities.-- The members of Group I showed a strong interest in athletics, and some were varsity players and managers of football, baseball, and basketball. Others belonged to the school sailing club, and some boys and girls owned their own boats and sailed in the annual regattas. Swimming was another almost general sport. Other activities were ice skating, roller skating, skiing, bowling, horse-back riding, bicycling, dancing, boxing, camping, fishing, badminton, archer, and tennis.

Pupils in this group held membership in the student council, library staff, school newspaper staff, rifle club, cafeteria squad, traffic squad, orchestra, band, glee club, Spanish club, stamp club, and motion picture operator's club.

Out-of-school activities included membership in Boy Scouts, Camp Fire Girls, Youth Canteen, Catholic Youth Organization, DeMolay, Rainbow Girls, Civil Air Patrol, Wing Scouts, American Legion Band, and Junior Achievement Companies.

One of the youngest boys included as an activity, "making the girls angry". The boy with the intelligence quotient of 165 was the only one to mention that he liked to read dog stories. The oldest boy played pool, attended movies often, played basketball, bowled, and wrote poetry. The son of the loftsman liked playing with all kinds of motors, pullies, derricks, and levers. A member of the rifle club expressed his chief interest as "anything to do with guns".

Pupils in Group II had many interests in common with those in Group I. In addition, the group included a girl gifted in drawing and in dress designing, several choir boys and altar boys, Sea Scouts, a hospital-aide, tennis players, several cartoonists, an amateur photographer, and a maker of model airplanes. One boy played the drums with a seventeen-piece band evenings in company with former members of Guy Lombardo's and Artie Shaw's bands. He collected pictures of drummers and took pride in studying with a former teacher of Gene Krupa. He had no use for geometry and was taking the course only because his father insisted that he should. With the exception of this boy, the extra-curricular and out-of-school activities of the pupils were an asset rather than a detriment to their work in the unit. Boys and girls who had as many and as varied outside interests as these pupils had possessed the initiative and drive essential to the successful pursuance

of the unit method.

Colleges for which pupils were preparing.-- Only twelve pupils of the total fifty-three did not plan to attend college. Pupils in Group I were interested in from one to three each of the following colleges: Amherst, Boston College, Boston University, Colby, Columbia, Dartmouth, Emmanuel, Georgia Technical Institute, Holy Cross, Jackson, Massachusetts Institute of Technology, Mount Holyoke, Northeastern, Notre Dame, Purdue, Radcliffe, Regis, Simmons, Tufts, University of Alabama, University of Southern California, and the United States Naval Academy.

Group II added to the above list: Harvard, Lehigh, Massachusetts School of Art, University of Minnesota, United States Coast Guard Academy, Wentworth Institute, Wellesley, West Point, and business colleges.

## The School

Situation of the school.-- The school in which the unit was to be taught is one of two high schools in an historic city of over eighty thousand population just south of Boston. It is a six-year high school of approximately eighteen hundred pupils and seventy teachers. The main building was erected in 1927 and has had two large wings added since then. The light brick building faces a large lawn surrounded on two sides by sloping grounds planted with flowering shrubs.

The northern part of the city in which the school is located includes attractive residential sections, an extensive beach which is part of the Metropolitan District Commission, two yacht clubs, a famous golf course, a large Naval air base, and a small historical park on a peninsula overlooking Boston. Also, within the school boundaries are two commercial centers each of which has a branch office of the two largest banks in the city, theaters, real estate and insurance offices, and fine stores.

There is a diversified industrial development in the northern part of the city, although the granite business and the shipbuilding plants, the city's most famous industries, are located in other parts of the city. Instead, there are the following industries, many of them also nationally known: iron foundry, pneumatic scales company, telephone apparatus factory, gear works, paint and varnish factories, lumber companies, yacht yard, rope factory, milk firm, cymbal works, stud and rivet factory, and the original store of what has grown to be a famous national chain of restaurants and ice cream stands. For the most part, the industries are housed in well-built factories with landscaped grounds.

The classroom.-- The classroom, twenty feet by twenty-eight feet is on the fourth floor of the building and has a pleasant southwest exposure overlooking the Blue Hills and the marshes of the Neponset River. Low flying planes

from the Naval Air Station, Navy blimps, and an occasional flight of sea gulls inland from the bay were part of the outside setting.

The furniture was movable, in units of a desk with chair attached, and while usually lined up in rows, could be arranged in groups of two's, three's, or four's for group work. When the pictures of the room were taken to illustrate the unit, the pupil photographer had the desks arranged so as to get as many pupils as possible into the picture. In the actual teaching of the unit they were arranged so that fire laws were observed at all times, that is, so that a passageway was left all around theroom and no exits were blocked. The room had two doors at one side opening into the corridor, one at the front facing a landing at the head of a stairway, a swinging door at the rear opening into the next classroom, and another door at the back leading into the book closet.

Other furniture in the room consisted of the teacher's desk and chair, a circular library table with three chairs, a glass-fronted built-in bookcase, and a movable bookcase which had been built in the school shop.

The movable furniture was well adapted to the laboratory phase of the unit. The book closet provided a good storage space for tests and illustrative materials, and the movable book case was useful to hold reference books.

The outstanding drawback was the makeshift nature of the bulletin boards. Hanging at the back of the room was a burlap covered, plywood bulletin board, twenty-seven inches wide and forty-six inches long which the teacher had had made when she first was assigned to the room. At the front of the room over the section of blackboard nearest the window was a plyboard bulletin board twenty-eight by thirty-six inches. It was attached to the board by hooks so that it could be removed when the blackboard was needed for other purposes. This bulletin board had been obtained from the biology teacher across the hall by trading her a three-winged, three-foot high, standing display of blood transfusions and serum preparations, which the son of a druggist had donated to the mathematics teacher. She had planned to paint it and use it as a standing screen on which to mount mathematical displays, but to the biology teacher this was a waste of good material, so it was traded to her for one of the plyboard bulletin boards in her room. Before the unit was finished, the teacher had obtained large, dark red cardboard sheets and had attached them by Scotch tape to the rear wall and had allowed pupils to attach papers to them and directly to extra blackboard spaces. Part of the "pooling-of-experiences" phase of a unit in mathematics consists of display of the work of various groups, so bulletin boards are important.

There was a pleasant solution to the bulletin board problem the September after the second unit had been completed. During the summer vacation the school department had had a sheet rock bulletin board constructed and attached directly to the rear wall of the room. It was eight feet long and four feet wide with a four-inch wooden frame. Moreover, there were similar ones in each of the other three rooms of the senior high school mathematics department.

In lieu of a metal filing case, the teacher used heavy cartons, nine inches high, eleven inches wide, and thirteen inches long, which were donated by a grocery store. Material was filed edgewise, but the tops were left on the cartons and fastened back so that material could be packed flat and covered when the unit was finished.

The other furnishings of the room were those common to mathematics classrooms: a hanging blackboard for graphs, board protractor, compasses, straight-edges with handles suspended from hooks under the chalk tray, and celluloid isosceles-right triangles and 30-60-right triangles which were suspended from hooks in upper frames of the blackboards.

Books.-- At the beginning of the first unit each pupil had a copy of the textbook Modern School Geometry.[1/]

1/John R. Clark, Rolland R. Smith, Raleigh Schorling, Modern School Geometry, World Book Company, Yonkers-on-Hudson, New York, 1943 Edition.

In the open bookcase was one copy each of the books listed as R-1 to R-5 in "Pupil References". (Page 45). The third book was school property. The fourth reference book had been purchased by the teacher, and the rest were sample copies given by book agents. In the glass-fronted bookcase were the books on the history of mathematics and other books meant to be used for the optional related activities. These were the property of the mathematics department and were shared by other teachers. On the table were copies of "The Mathematics Teacher",[1] the property of the teacher.

An encouraging aspect of the unit was the personal approval given by the superintendent of schools to a request for more reference books. Before the end of the school year, the books began to arrive, and by the next September the twenty-six requested books were ready for use.

New reference books.-- The new books which arrived in time for use the next year were additional copies of the reference books listed in the pupils' references on page 45. Specifically, there were five copies of Basic Mathematics, Betz; five of Plane Geometry, Keniston and Tully; one copy of Plane Geometry and Its Uses, Mirick, Newell, and Harper; and five copies each of Plane Geometry, Revised and Enlarged Edition, Morgan, Foberg, and Breckenridge; Today's Geometry, Revised Edition, Reichgott and Spiller;

1/National Council of Teachers of Mathematics, New York City, 27, Issues of the Mathematics Teacher.

Plane Geometry, Revised Edition, Welchon and Krickenberger.

Preparation of the Unit

Determination of objectives.-- Similar triangles was one of the topics in the course of study prescribed for the tenth-grade geometry course. To adapt the topic to the unit method, the outline form of the course of study was changed to a more detailed treatment in which the objectives were changed from the infinitive or participial form to a series of declarative sentences. From a study of this new list of objectives a general statement of the unit was written. This statement encompassed the main purpose to be accomplished. Then there was written the delimitation of the unit, which, as in the revised outline of the course, consisted of declarative sentences stating exactly what was to be taught. There was also a list made of probable incidental and indirect learning products. The unit of experience sequence or unit assignment was then prepared. (Pages 30 to 40) This incorporated the introductory activities, core-activities for which every pupil would be held responsible, laboratory experiences, optional related activities, predetermined instruction points, intervals for pooling-of-experiences, and a tentative time for the evaluation of the unit. For the pupils' use there was prepared a pupils' study guide which contained the core material with instructions for group or individual work

and with detailed references to the textbook and to reference books. (Pages 41 to 57)

In the unit method each pupil should have his own mimeographed copy of the study guide and of the reference books. However, in this unit, the pupils in the commercial department were busy helping so many other departments that the mathematics teacher was content to have their assistance in preparing copies of the examination for the unit. She used the typed copies that her student secretary prepared of the other material. The typed copies were posted on the bulletin board, and pupils took notes from them during the progress of the unit. Optional related activities should be typed on library cards and filed for pupil reference. In the first unit, however, a typed list was posted on the bulletin board. As can be seen, the first unit illustrates an attempt to incorporate as many aspects of the unit method as conditions permit even though at first it may not be possible to use the method in its entirety.

Preparation of the examination.-- The examination consisted of two sections to be given on successive days. Section A contained seventy true-false statements and fifteen items about steps in the proofs of exercises. The true-false statements tested the understanding of concepts and the ability to apply them in computation, and followed

the arrangement of items in the delimitation of the unit. Section B of the examination required the numerical solution of twenty-one exercises about similarity, and was also arranged to test each item of the delimitation of the unit.

Section A, as has been indicated, was prepared in mimeograph form and stapled by pupils in the commercial department. Six stencils were required. The students who did the work and the office practice teacher who supervised it were deserving of praise for the fine way in which the work was done. Since the commercial pupils were unfamiliar with some of the mathematical symbols and notation, and since they had to use the mimeoscope to draw the geometric figures, the work took longer than the estimated time. If the examination had been ready for the first day of the unit, it would have been given as a preliminary test so that pupil progress in the unit could have been measured. This is a feature of the unit method and was followed in the second unit. In the first unit, then, there was no pre-test given.

Introduction of the unit.-- At the time the unit was being prepared the activities of the committee appointed to select a site for headquarters of the United Nations were being publicized in the Boston and local newspapers. The teacher collected clippings referring to the Blue Hills plot of land which was one of the locations under consideration. There was in the possession of her family and

available for her use a blue-print, thirty-four inches by twenty-one inches, of land in and adjacent to, the Blue Hills Reservation. It had been prepared by the city engineer and was drawn to the scale 1 inch = 300 feet. To introduce the unit the blueprint and newspaper clippings were mounted on the bulletin board in the back of the room.

Summary of preparations.-- A last check on preparations showed that the examinations were being prepared in the commercial department. The typed copies of pupils' activity guides, optional related activities, and pupils' references were displayed on the small bulletin board at the front of the room. The reference books were arranged in the open-shelved bookcase. The teacher's copy of experience sequences was available for reference, and the introductory bulletin board display had been mounted. In addition, preparation was made to keep a record of important occurrences in a log book. As the unit progressed, entries would be made of significant happenings. At the conclusion of the unit the entries indicated those phases of the unit which should be improved or eliminated and those which were worthy of being retained.

Table 4. Pupil Scores on Final Examination

on Similar Triangles

Groups I and II

| Pupil | Score in Section A | Score in Section B | Total Score |
|---|---|---|---|
| 1 | 59 | 13 | 72 |
| 2 | 59 | 9 | 68 |
| 3 | 69 | 13 | 82 |
| 4 | 72 | 14 | 86 |
| 5 | 60 | 9 | 69 |
| 6 | 47 | 7 | 54 |
| 7 | 61 | 17 | 78 |
| 8 | 31 | 5 | 36 |
| 9 | 64 | 17 | 81 |
| 10 | 61 | 10 | 71 |
| 11 | 47 | 12 | 59 |
| 12 | 51 | 7 | 58 |
| 13 | 49 | 11 | 60 |
| 14 | 52 | 10 | 62 |
| 15 | 71 | 9 | 80 |
| 16 | 46 | 8 | 54 |
| 17 | 70 | 15 | 85 |
| 18 | 49 | 7 | 56 |
| 19 | 57 | 10 | 67 |
| 20 | 70 | 10 | 80 |
| 21 | 46 | 9 | 55 |
| 22 | 51 | 7 | 58 |
| 23 | 35 | 4 | 39 |
| 24 | 47 | 12 | 59 |
| 25 | 59 | 8 | 67 |
| 26 | 57 | 8 | 65 |
| 27 | 58 | 12 | 70 |
| 28 | 52 | 11 | 63 |
| 29 | 56 | 9 | 65 |
| 30 | 61 | 14 | 75 |
| 31 | 52 | 6 | 58 |
| 32 | 57 | 8 | 65 |
| 33 | 68 | 12 | 80 |
| 34 | 50 | 5 | 55 |
| 35 | 61 | 6 | 67 |

Table 4. (Concluded)

| Pupil | Score in Section A | Score in Section B | Total Score |
|---|---|---|---|
| 36 | 64 | 9 | 73 |
| 37 | 69 | 10 | 69 |
| 38 | 25 | 9 | 34 |
| 39 | 35 | 9 | 44 |
| 40 | 44 | 5 | 49 |
| 41 | 62 | 10 | 72 |
| 42 | 55 | 8 | 63 |
| 43 | 58 | 12 | 70 |
| 44 | 52 | 11 | 63 |
| 45 | 39 | 5 | 44 |
| 46 | 53 | 8 | 61 |
| 47 | 53 | 9 | 62 |
| 48 | 61 | 16 | 77 |
| 49 | 43 | 8 | 51 |
| 50 | 54 | 8 | 62 |
| 51 | 45 | 9 | 54 |
| 52 | 43 | 3 | 46 |
| 53 | 47 | 6 | 53 |

# CHAPTER II

## UNIT ORGANIZATION OF THE TOPIC

## SIMILAR TRIANGLES

### General Statement of the Unit

General statement.-- Similar triangles involve abstraction and realia. Their treatment in the tenth-grade course in geometry exemplifies deductive reasoning, and the principles thus derived are employed to solve practical problems. In this unit the logical treatment stems from the definition of similar polygons and branches out to theorems which provide shorter methods of establishing the similarity of triangles. Problems of indirect measurement are solved by proportions derived from the corresponding sides of similar triangles or by theorems, the proofs of which depend upon similar triangles. One of the theorems regarding similar triangles supplies a short proof of the Pythagorean theorem.

### Delimitation of the Unit

1. A map of Massachusetts and the state of Massachusetts have the same outline or shape. The map is a reproduction of the real boundary lines of the state. Also, a blueprint of a section of

land shows in miniature the shape of the actual section of land. These are illustrations of similar figures.

2. By means of similar figures any figure can be reproduced on an enlarged or diminished scale.
3. If polygons have the same shape, even though they differ in size, they are said to be similar.
4. The definition of similar polygons is: Similar polygons are polygons whose corresponding angles are equal and whose corresponding sides are in proportion.
5. The symbol for <u>similar to</u> or <u>is similar to</u> is $\sim$.
6. Since a definition is reversible, it follows that if polygons are similar, their corresponding angles are equal and their corresponding sides are in proportion.
7. The factor of proportionality is a number which tells that the sides and diagonals of one polygon are that many times as long as the corresponding sides or diagonals of the similar polygon.
8. If polygon ABCDE is similar to polygon A'B'C'D'E', and k is the factor of proportionality, then

$\frac{AB}{A'B'} = k$, $\frac{BC}{B'C'} = k$, $\frac{CD}{C'D'} = k$, $\frac{DE}{D'E'} = k$, and $\frac{EA}{E'A'} = k$.

Notice that this may be expressed as $AB = k.A'B'$, $BC = k.B'C'$, $CD = k.C'D'$, $DE = k.D'E'$, and $EA = k.E'A'$.

9. When geometric figures are congruent, the ratio of corresponding sides is one. This means that the factor of proportionality is one.
10. A special study is made of similar triangles.
11. Everything said concerning similar polygons applies to similar triangles with the exception of diagonals.
12. In the case of two triangles the angles of one cannot be equal to the angles of the other without the corresponding sides being in proportion.
13. Conversely, the corresponding sides of two triangles cannot be in proportion without the corresponding angles being equal.
14. There are three theorems by means of which triangles can be proved similar.
    a. Two triangles are similar if two angles of one are equal respectively to two angles of the other. A corollary is: Two right triangles are similar if an acute angle of one is equal to an acute angle of the other. To prove the theorem we show that the definition of similar polygons applies. Another corollary is: Corresponding altitudes of similar triangles have the same ratio as any two corresponding sides.

b. Two triangles are similar if they have an angle of one equal to an angle of the other and the sides including these angles are in proportion. This theorem can be proved by means of the first one.

c. Two triangles are similar if their corresponding sides are in proportion. The proof involves the second theorem in similarity and one on the congruence of triangles.

15. Proportions derived from the corresponding sides of similar triangles are of great value in determining inaccessible distances.

16. Two theorems about circles are useful in providing proportions to solve practical problems. Their proofs follow the chain of reasoning started with the definition of similar polygons.

a. If two chords intersect within a circle, the product of the segments of one equals the product of the segments of the other.

b. If from a point outside a circle a secant and a tangent are drawn, the tangent is the mean proportional between the secant and its external segment. A corollary to this is: If from a point outside a circle two secants are drawn, the product of one secant and its external segment is equal to the product

of the other secant and its external segment.

The first of the two circle theorems is useful to work out a "rule of thumb" whereby workmen can find the center of a broken gear when they have only a part of the gear and do not know the radius. The second provides a formula applicable to estimating distances at sea from the deck of a vessel to another ship visible on the horizon.

17. Two other important theorems are:

a. In any right triangle the altitude upon the hypotenuse is the mean proportional between the segments of the hypotenuse.

b. If the altitude upon the hypotenuse of a right triangle is drawn, either side is the mean proportional between the hypotenuse and the adjacent segment.

The first is useful in constructing the mean proportional to two given line segments, and the second proves the theorem of Pythagoras.

This unit is to be followed by a unit on numerical trigonometry. Areas of similar triangles are left to be studied in the unit on area of polygons. Other theorems and exercises regarding similar polygons are treated in grade eleven.

List of Probable Indirect and Incidental

Learning Products

Inductive reasoning proceeds from particular cases to the general law.

Deductive reasoning starts with the general law and applies it to a particular case.

Deductive reasoning is used in demonstrative geometry.

When the theorems in a particular field are known, a problem may be solved by classifying it as a special case of one of the theorems.

A "rule of thumb" requires a good memory only. The person who understands the underlying principle can work out his own method of solution in case he has forgotten the rule of thumb.

Pupils who have trouble with the abstractions of geometry can learn the practical part of the subject in a different course.

Those who can solve the more difficult exercises and who enjoy this kind of work should plan to continue in the sequential college-preparatory-mathematics courses.

Proofs of similar triangles depend upon the new definition, former definitions, postulates, and theorems of ratio and proportion, and in one case, one of the theorems of congruent triangles.

With the exception of loci, this is the first unit

of work in geometry so far which has not started with at least one new postulate.

This unit is necessary if we are to understand the following work on numerical trigonometry.

A geometric principle, such as the theorem of Pythagoras, can be used long before it is fitted into the chain of reasoning. It may, first of all, be accepted merely on the word of a teacher and because it works in practice. It affords real satisfaction, however, when we see later how it can be proved deductively.

There are similar cases, as the above, in other phases of life. One may not always understand the "why" of certain home, school, or social restraints or customs. It is a good idea to conform to them, realizing that they are based on reasons which will become clearer as one learns more of life.

It pays to be accurate in the algebra or arithmetic involved in exercises. A good plan based on the correct principle in geometry can be spoiled by carelessness in computation.

Different textbooks use different orders of postulates and theorems. "The correctness of a proof or of any other course of reasoning cannot be judged independently of the assumptions upon which it rests".1/

1/George David Birkhoff and Ralph Beatley, Basic Geometry, Scott, Foresman and Company, New York, 1941, p. 39.

It is important to stand well and to speak in a clear voice when proving a theorem or an exercise in geometry. You cannot convince a fellow student if he cannot understand what you are saying.

It is courteous to let a student conclude his proof before you comment on it. If you detect an error or have something to add, wait until he has finished.

Group work ends when a test begins. A student is then strictly on his own.

There is a profound sense of satisfaction that accompanies the successful solution of a difficult exercise in geometry.

".... a student who habitually disciplines himself to keep his thoughts in order is much more likely to do himself justice in an emergency; and there is an intimate association between orderly writing and orderly thinking."[1]

1/College Entrance Examination Board, Specimen Answers of College Candidates in Plane Geometry, Document Number 99, The Board, New York, 1921.

Unit Assignment

Tentative Time--Three and One-Half Weeks

1. Introductory activity.-- Bulletin-board display and discussion conducted by the teacher. The display consists of a large blueprint of the Blue Hills section of Quincy and newspaper clippings revealing the continuation of the dispute regarding the United Nations site. One headline of the local paper is "Blue Hills Still Have Chance as U.N. Site". Part 1 to 3 of the delimitation of the unit will be developed through this discussion. Naturally, this method of introducing the unit must be changed if the unit is used another time. In the absence of a pertinent, timely introduction, maps of various sizes may be used.

   Core activities.-- Those marked with an asterisk appear on the pupils' study guide.

*2. The statement that similar polygons are polygons with the same shape but not necessarily the same size is merely a description of similar polygons.

   a. What is the definition of similar polygons as given in the study of geometry? Consult your textbook or any book on the reference shelf. Use the index. Note the name and author of the book you use.

   b. Find the abbreviation of similar to or is similar to.

c. Compare the new symbol with the symbol for <u>is congruent to</u>.

d. Remember that a definition is reversible. Give the reverse of the definition of similar polygons. (T refers to the textbook; R-1, R-2, and such, refer to reference books.) See Pupils' References. (Page 45).

*3. Group work. Each member of the group may assume responsibility for a certain number of exercises or questions. Try to avoid much duplication within the various groups. Prepare this work for class demonstration.

T: 323:1-9

R-1: (Easy)
220-222
368
374-377

R-2: None--Consult the book and see why there is no assignment.

R-3: 263:1-7

R-4: 227:Read
228:1-4

R-5: 162:40-41

Group leaders will be in charge of class demonstration.

4. Discussion conducted by teacher, with pupils referring to textbooks or reference books.

a. How are similar triangles easier to work with than similar polygons of more than three sides?

b. Show how the three theorems simplify the establishment of similarity of triangles.

c. Remind pupils of the necessity of proving these theorems.

d. Develop the proof of theorem 54 with the class. (Theorem numbers are those used in the textbook.) Encourage pupils to compare the proofs given in the reference books.

*5. T:324. Prepare the demonstration of theorem 54. Answer the following questions concerning the proof:

a. What former theorem and corollary appear as reasons in this proof?

b. What statements apply to the first part of the definition?

c. What statement clinches the second part?

d. Prove the corollary concerning right triangles.

e. T:332. Prove the theorem concerning the altitudes of similar triangles.

*6. Class work. Demonstration of theorem 54 and corollaries.

*7. Group work. T: 325:1,2,5,6,9. Choose any other two exercises.

8. Teacher develops proof of theorem 55 to show how the proof depends upon theorem 54 and show how it solves exercises 1-3.

The only pupils held responsible for this proof are those who are preparing to enter colleges with rigid requirements or pupils who are apt to become candidates for scholarships.

*9. From the following choose ten exercises including at

least one you consider easy and at least one you consider difficult. Remember the reverse of the definition. Also remember that to find corresponding sides, look opposite equal angles.

R-2: 202:1-2 (Pages not the same in new edition.)

R-3: 265:1-6 268:1
266:7-15 269:2-10
267:16-24

R-4: 229:3,4,6. Shadow problems. How do you account for the corresponding angles being equal in such problems?

R-5: 166:57-59. Note the shadow problem.

Prepare a neat presentation of the exercises you have chosen. Each group may choose one representative to present the most difficult exercise solved by that group.

10. Discussion of theorem 56, comparison of the theorem and the definition of similar polygons, and development of the proof. Note that the approach used in theorems 54 and 55 will not work since no angles are given equal. Have the class try to suggest other possible attacks, referring to the book only as a last resort. Demonstration of this theorem is not required for all; but all pupils should be able to follow the proof when it is presented by the ablest students.

*11. Groups report to class on assignments 9 and 10.

*12. Remember that you now have four ways of proving triangles similar, three theorems and a corollary. Be sure that

you can prove theorem 54 and can follow the proofs of theorems 55 and 56 when they are given as special topics.

13. Check up on setting up proportions and solving for the missing term in numerical exercises.

*14. Can you find any new information telling how to prove four lines proportional? Do not forget theorem 57. Make a list of any new methods or illustrations that you find. T:328. Consult as many reference books as you can. Use the table of contents and index.

*15. Exercises proving triangles similar and lines proportional. Group work.

T: 329:1-9
330:10-19

R-2: 203:1-3
205:5-16
206:17-25 Pick an easy one and one that looks hard.

R-3: 271:1-6
276:1-2 top
1-3 bottom
277:4-13
278-282:14-18
1-13
1-6

R-4: 229:7 A good practical one.
230:8 If you are curious about the "grade" of a road, see this one.
230-231 Camera fans, see these pages.

R-5: 167:61
168:62-65
169:66-67
291:28-31 Make a special effort to do at least one of the above.

In organizing group work on the above, try to assign for class presentation at least one exercise typical of the various kinds you find.

*16. Quiz on proving triangles similar, setting up proportions, and solving for a missing term.

*17. How can you prove that the product of two segments is equal to the product of two other segments? See T:331. Why is it desirable to do this? Group work.

T: 331:1-6

T: 332:1-2

R-1: 279:1-8
280:9-16 Theoretical situations.
281:17-23

428:36-37
429:38-41 Actual situations. Prepare a good illustration for these and explain carefully how the distances are laid out and measured. Arrange with group chairmen so that all of these exercises are done.

*18. Here is a problem sent in by the father of a boy in the fifth-period class. It can be solved by using theorem 58. Before you look up the theorem, see if you can solve the problem by your present knowledge of similar triangles. Then see how theorem 58 simplifies the solution.

A section of a broken gear is sent to a shop to be a pattern for a new gear of the same size and shape. However, the section sent does not disclose the radius. How may the radius be found? What measurements can be

taken from the sample that will help to find the radius? When you have worked it out, see the clipping from Life magazine and compare your solution with it.

See R-3: 318:4 for this same type question in regard to a railroad curve.

*19. Prepare to demonstrate theorem 58.

*20. Group work.

T: 333:1-8 R-4: 259:1-4

R-2: 222:1-4 R-5: 171:79-80

R-3: 297:1-8

*21. Prove theorems 59 and 60.

T: 334-335

*22. T: 334:1-9. Choose three, two numerical and one literal.

T: 335:1-9. Choose three, one from 1-4, one from 5-7, and either 8 or 9.

R-3: 299-301. Choose one from each page.

R-3: 303. Practical problems. Choose as follows: 1 and 6 for camera fans; 4 for Boy Scouts or future surveyors; 5, a shadow problem at night, well worth investigating.

Here are some harder ones. Try to do at least three.

R-2: 223:1-3

R-4: 260:1-6 R-5: 173:89-100

*23. Quiz on work in 17-22 inclusive.

*24. The pilot of a plane observes that he is 600 feet above the ground. How many miles ahead can he see on the

earth's surface?

See if you can find a previous theorem which will apply here. If necessary, consult R-3: 430:42. Prepare an exhibit based on this explanation and on R-3: 430:43-46. For a very good picture of this see R-4:261. Note that this merely tells you the special formula, while R-3 derives it from the theorem. Include the following exercise in your report. Solve it both ways and see whether there is any appreciable difference in the results.

The eye of an observer at sea on the deck of a vessel is 40 feet above the water line. The entire hull of a boat has just become visible on the horizon. Assuming the diameter of the earth to be 8000 miles, how far away from the observer is the boat?

25. Test pupil skill in solving quadratic equations before starting the next geometric concepts.

*26. Board work to correct general weaknesses disclosed by pre-test on quadratic equations.

T: 336:1-14
338:1-5; 11-15; 22-29

*27. Theorem 61, page 339, is considered by some to be second in importance to the Theorem of Pythagoras in its applications. See how it works before proving it.

T: 340:2,8

R-2: 183;144,145

Now prove theorem 61 by using similar right triangles and setting up the proportion which uses the sides mentioned in the conclusion. Mark the equal angles so as to be sure to find corresponding sides.

*28. Construct the mean proportional to two given line segments by means of theorem 61. See T: 340 or R-4: 219.

*29. The Theorem of Pythagoras, which you have been using since the eighth grade can now be fitted into the chain of reasoning. Then you proved it inductively. You constructed right triangles on squared paper, making the legs 3 and 4 units, or 5 and 12 units and then found 5 or 13 units for the hypotenuse. You then observed the following: $3^2 + 4^2 = 9 + 16 + 25$, which equaled $5^2$; or $5^2 + 12^2 = 25 + 144 = 169$, which equaled $13^2$.

Now we are ready to prove the theorem deductively by using theorem 62. Prove theorem 62 first, using the plan we used for theorem 61.

*30. Now prove theorem 63, which is the Pythagorean Theorem. Make a neat copy of the formal proof for display on the bulletin board. You may consult R-5: 283-284.

*31. T: 343:1-5. Choose one
6-8. Choose one.
9-12. Choose one.

*32. Give to radicals the same treatment that was given to quadratic equations. Use T: 344-345.

*33. Quiz on quadratic equations and on simplifying radicals.

*34. There are two special right triangles used by carpenters and workers of various kinds, the isosceles-right-triangle and the 30-60-right triangle. They are used by us in our own work so much that we note the relations of one side to the others and record them in a table. After a while, most pupils find that they have used them so often that they have memorized them and can discard the table. You may not use the table in an examination. These relations may be obtained by working them out again any time you need them, but it is a great time-saver to memorize them. See T:345-347 for help in making the table.

*35. Here are exercises requiring the use of the Theorem of Pythagoras or one of the special applications. Do twenty exercises each. Keep two separate sections in recording your work:

| Section I | Section II |
|---|---|
| Problems involving real situations. | Problems involving right triangles with no special application given. |

Try to find more exercises for Section I than for Section II.

T: 347:1-6
348:7-17

R-2: 176-178

R-3: 284-289 Any exercises. Note the bicycle question on 288.

R-4: 63:1-2 Every one here is a real problem.
64:3-9
65:1-8 Note the tent problem.
90:19

*36. Quiz on the use of the Pythagorean Theorem, the 30-60-right triangle and the isosceles-right-triangle.

*37. In preparation for the final examination try yourself out on the following examinations:

T: 350 Chapter Test

R-2: 208-209 Test VII

R-5: 302 Completion Test

*38. An objective test including questions on all the definitions, theorems, constructions and applications of the above.

General Study and Activity Guide

Pupil's copy.-- The pupil's copy of the General Study and Activity Guide consisted of eleven typed pages. Since this material was identical with the items marked with asterisks in the Unit Assignment, it will not be repeated here.

Optional Related Activities

1. Try to obtain from the Squantum Naval Air Base copies of aerial photographs which they may be willing to release.
2. See if George Lawley and Sons have any blueprints of ships which they will give you.
3. Try to obtain blueprints from Fore River Shipyard, Boston Gear Works, Pneumatic Scale, Norfolk Iron Works, or any other local concerns.
4. See if you can obtain part of a broken gear, including an arc.
5. Prepare a display of floor plans and pictures of rooms or houses. Suggestions: Royal Barry Wills, or a similar book on home planning; material given by Quincy Savings Bank; recent copies of Ladies' Home Journal, Good Housekeeping, Woman's Home Companion, or such magazines.
6. Make an exhibit showing how similar triangles occur in photography, or their application to the camera eye. Consult the biology teacher and R-4: 230-231.
7. Show a method for determining the radius of the earth. See R-2: 240:5. Test this method and write up an account of it, including the measurements and computation.
8. Measure the height of the flagpole on the school ground.

9. Investigate similar triangles as applied to aerial photography. See R-4: 239-240. Using the camera dimensions and ground line given, figure the approximate altitude at which the photograph of Washington, D.C., was taken. Arrange your work so that it may be displayed on the bulletin board.
10. Thales is called "The First of the Seven Wise Men of Greece". See "The Mathematics Teacher", Vol. XXIII: No. 2, for a picture of Thales. Read the article to see why he is mentioned in a study of similarity. Some "artist" in the class can probably make a good copy of the picture of Thales. For further information consult "The History of Mathematics". (Pupil References)
11. See "Originals in Plane Geometry for College Board, Annapolis, and West Point Entrance Examinations" by Horblit for special problems if those assigned seem too easy. (Pupil References)
12. The Greeks believed that a certain ratio of the width of a rectangle to its length produced the most pleasing rectangle. It is called the "Divine Proportion" or the "Golden Section". Consult Clark-Otis, Modern Plane Geometry, R-3: 431-432 (an easy explanation), or R-5: 256-257, a more difficult description, but one which some will find challenging.

13. Continuation of 12: Test the dimensions of some rectangles by the Golden Section. Try our school room, our bulletin board, a door or window.

    The dimensions of the East Room in the White House are supposed to contribute to the beauty of the room. Look them up in a guide book of Washington. What is the shape of this room? Try to find a picture of it and report on any measurements of special interest.

14. For pupils interested in military applications of similar triangles: Ask your instructor for "General Survey of Illustrative Military Applications in Elementary Mathematics". (Pupil References). See the aerial photograph M-22 and problems 12-17.

15. Construction of scales: Aviation Mathematics, Buchan, Borthwick, Wodden: 41. Prepare an illustrated report.

16. Pythagoras should not be slighted. See item 10 for instructions on how to investigate his work. R-2: 174.

17. Geometric proof of the Theorem of Pythagoras:

    R-2: 173-174 (Note Raphael's conception of Pythagoras) 213-214 in the new edition.

    R-3: 332-333

    R-5: 296--Note that this proof is attributed to Euclid. Pythagoras' own proof is unknown.

18. Study President Garfield's proof of the Pythagorean Theorem: R-2: 215 in the new edition, R-3: 217, R-4: 175.

19. Note the existence of a number of alternative proofs of the Pythagorean Theorem. See how many you can find.
20. Look up the use of similar figures for reducing or enlarging drawings.
21. Suggest any other activity concerning similar triangles that you consider worth investigating.

## Pupil References

The books included in this reference list are arranged as in a bibliography with the addition of such notations as T and R-1. In the copy which was given to the pupils only the notation, the name of the book, and the name of the author were given. Books with no notations were put at the end of the list when it was prepared for the pupils. Such books were of general interest or were especially reserved for optional related activities.

The notations T, R-1, R-2, and the like, conform to the notations used in the unit assignment and in the pupil study guide. They refer to the textbook used by the class and to the respective reference books.

R-1, Betz, William, Basic Mathematics, Ginn and Company, Boston, Massachusetts, 1942, pp. 116-135, 220-22.

Buchan, A.F., R. Borthwick, William R. Wadden, Aviation Mathematics, Houghton Mifflin Company, Boston, Massachusetts, 1942, pp. 39-42.

Cajori, Florian, A History of Elementary Mathematics, The Macmillan Company, New York, 1929, pp. 48-53.

T, Clark, John R., Rolland R. Smith, Raleigh Schorling, Modern-School Geometry, Revised, Schorling-Clark-Smith Series, World Book Company, Yonkers-On-Hudson, New York, 1943, pp. 321-350.

Horblit, Marcus, Originals in Plane Geometry for College Board, Annapolis, and West Point Entrance Examinations, Marcus Horblit, Cambridge, Massachusetts, 1942, pp. 52-64.

R-3, Morgan, Frank M., John A. Foberg, W.E. Breckenridge, Plane Geometry, Houghton Mifflin Company, Boston, Massachusetts, 1943, pp. 263-281.

National Council of Teachers of Mathematics, "Pythagoras", The Mathematics Teacher, (March 1930), 23:3, Frontispiece and pp. 185-186.

_____, "Thales", _____ (February 1930), 23:2, Frontispiece and pp. 84-86.

R-4, Reichgott, David, Lee R. Spiller, Today's Geometry, Revised Edition, Prentice-Hall, Incorporated, New York, 1944, pp. 226-238.

Sides, Winfield M., General Survey of Illustrative Military Applications in Elementary Mathematics, Department of Mathematics, Phillips Academy, Andover, Massachusetts.

Smith, David Eugene, History of Mathematics, Volume I, Ginn and Company, Boston, Massachusetts, 1923, pp. 69-77.

R-5, Wells, Webster, Walter W. Hart, Modern Plane Geometry, D. C. Health and Company, Boston, Massachusetts, 1926, p. 151-178, 185-186.

Other Books Which Were Ordered and Were

Obtained at the Close of the Unit

Hart, Walter W., Basic Mathematics, D. C. Heath and Company, Boston, Massachusetts, 1942, pp. 264-275.

Keniston, Rachel P., Jean Tully, Plane Geometry, Ginn and Company, Boston, Massachusetts, 1946, pp. 297-332.

Mirick, Gordon R., Marquis J. Newell, George A. Harper, Plane Geometry and Its Uses, Enlarged and Revised, Row Peterson and Company, Evanston, Illinois, 1935, pp. 247-263.

Seymour, F. Eugene, Paul James Smith, Plane Geometry, Publisher's Sample Copy, The Macmillan Company, New York, 1945, pp. 293-327.

Welchons, A. M., W. R. Krickenberger, Plane Geometry, Revised Edition, Ginn and Company, Boston, Massachusetts, 1940, pp. 345-363.

References for Teacher's Use

Beatley, Ralph, Coherence and Diversity in Secondary Mathematics, Eighth Yearbook of the National Council of Teachers of Mathematics, Bureau of Publications, Teachers College, Columbia University, New York, 1933.

_____, "Demonstrative Geometry", The Mathematics Teacher, 24: 213-222.

_____, "The Third Report of the Committee on Geometry", The Mathematics Teacher, (July 1935), 28: 401-450.

Billett, Roy O., Fundamentals of Secondary-School Teaching with Emphasis on the Unit Method, Houghton Mifflin Company, Boston, 1940, xxi and 671.

Perry, Winona M., "What are the Real Values of Geometry?" The Mathematics Teacher, 21: p. 51.

Pickett, Hale, An Analysis of Proofs and Solutions of Exercises Used in Plane Geometry Tests, Contributions to Education, Number 747, Bureau of Publications, Teachers College, Columbia University, New York, 1938, 122 p.

Reeve W. D. (Editor), The Teaching of Geometry, Fifth Yearbook of The National Council of the Teachers of Mathematics, Bureau of Publications, Teachers College, Columbia University, New York, 1930.

_____, "Bibliograpy on the Aims in Secondary Mathematics", The Mathematics Teacher, (Novemeber 1925) 18, 7.

_____, "Objectives in Teaching Demonstrative Geometry", The Mathematics Teacher, (December 1927) 20, pp. 435-450.

# GEOMETRY EXAMINATION

## SIMILAR TRIANGLES

### SECTION A

If the following statements are true, draw a circle around the number of the statement: O. If they are false, draw a cross through the number of the statement: X.

1. Similar polygons have the same shape but not necessarily the same size.

2. The map of a state and the actual boundary lines of the state form similar figures.

3. A figure can be reproduced on an enlarged or diminished scale.

4. An army hut 80 feet long would be 8 inches long if drawn to the scale 1 inch to 10 feet.

5. The northeast boundary line of Plot 6 in the Blue Hills neighborhood is 4474 feet. A blueprint scale 1 inch = 300 feet would represent this line by approximately 15 inches.

6. The definition of similar polygons is: Similar polygons are polygons whose corresponding angles are equal and whose corresponding sides are in proportion.

7. We do not know that triangles are similar unless we know that their corresponding angles are equal and their corresponding sides are in proportion.

8. All squares are similar.

9. All rectangles are similar.

10. Any rhombus is similar to any other rhombus.

11. All American flags are similar.

12. If two hexagons are similar, each angle of each hexagon will equal 120 degrees.

13. Any two congruent triangles are similar.

14. The converse of the above is true.

15. If two angles of one triangle equal two angles of another triangle, then their corresponding sides are in proportion.

16. Two right triangles are similar if an acute angle of one is 40 degrees and an acute angle of the other is 50 degrees.

17. In triangles ABC and A'B'C', if angle A equals angle A', AC equals 5 inches, AB equals 7 inches, and A'C' equals $2\frac{1}{2}$ inches, then A'B' equals $3\frac{1}{2}$ inches.

18. A triangle with two sides of 10 inches and 12 inches respectively, is similar to a triangle with sides of 15 inches and 18 inches respectively.

19. Two right triangles are similar if their legs are in proportion.

20. In similar triangles corresponding sides, altitudes, and medians are proportional.

21. Corresponding sides of similar triangles are opposite equal angles.

22. Two triangles are similar if their corresponding sides are parallel to each other or perpendicular to each other.

23. When the midpoints of the sides of a triangle are joined, the triangles formed are similar to each other and to the original triangle.

24. If three or more concurrent lines intersect two or more parallel lines, the corresponding segments of the parallel lines are equal.

25. The height of a tree may be measured by comparing the length of its shadow with the length of the shadow cast by an object of known height, parallel to the tree.

26. If a tower casts a shadow 84 feet long when a vertical 10-foot pole casts a shadow 7 feet long on level ground, the tower is less than 84 feet high.

27. Two chords AB and CD intersect at P. Triangle APC is similar to triangle DBP because angle APC equals angle DPB and angle A equals angle B.

28. If perpendiculars are dropped from points on the arms of an isosceles triangle to the base, similar triangles are formed.

29. A picture of a man standing 18 feet from a camera was 2 inches high. The focal length of the camera was 6 inches. This would show that the man was 6 feet tall.

30. If two chords intersect within a circle, the product of the segments of one is equal to the product of the segments of the other.

31. Chords AB and CD of a circle intersect at E. If AE = 8, BE = 3, CE = 4, then ED = $1\frac{1}{2}$.

32. The horizontal chord of a fan window is 4 feet and the perpendicular from its midpoint to the top of the window is 16 inches. The radius of the circle from which the window was cut was 3 feet.

33. If from a point outside a circle a secant and a tangent are drawn, the tangent is the mean proportional between the secant and the external segment.

34. If from a point outside a circle a secant and a tangent are drawn, the square of the tangent equals the product of the segments of the secant.

35. Given a circle with tangent AB and secant AC intersecting the circle at D. If AC = 20, AD = 5, then AB = 10.

36. The diametor of the earth is approximately 4000 miles.

37. A tangent 6 inches long can be drawn from a point 10 inches from the center of a circle. The radius, r, may be found by solving the equation: $(10+r)(10-r)=36$.

38. In exercise 37, the diameter, d, may be found by solving the equation: $d(10-\frac{1}{2}d)=36$.

39. If from a point outside a circle two secants are drawn, the product of one secant and its external segment is equal to the product of the other secant and its external segment.

40. If $X^2 = 25$, $x = 5$.

41. If $X^2 = 25$, $x = \pm 5$.

42. If $X^2 = 20$, $x = 2\sqrt{5}$

43. If $X^2 + 7 = 16$, $X = \pm 3$.

44. If $X^2 - 7 = 16$, $X = \pm\sqrt{23}$

45. In the expression $X^2 + 6X$, adding 9 to the expression will make it a perfect square.

46. The equation $x^2 - 4x = 45$ may be changed to the equation $x^2 - 4X + 4 = 49$.

47. In any right triangle the altitude upon the hypotenuse is the mean proportional between the segments of the hypotenuse.

48. If the altitude upon the hypotenuse of a right triangle is drawn, either arm is the mean proportional between the segments of the hypotenuse.

49. In right triangle ABC, CD is the altitude to hypotenuse AB. If AD = 9 and DB = 4, DC = 6.

50. In any right triangle the square of the hypotenuse is equal to the sum of the squares of the other two sides.

51. If c is the hypotenuse of a right triangle and a and b are the legs, then $b^2 = (c+a)(c-a)$.

52. $\sqrt{12} = 2\sqrt{3}$.

53. $\sqrt{12} = \pm 2\sqrt{3}$.

54. $\sqrt{2/3} = \frac{\sqrt{6}}{3}$

55. $\sqrt{2/3} = 1/3\sqrt{6}$.

56. $\sqrt{2/3} = \pm\frac{\sqrt{6}}{3}$

57. $\sqrt{657} = 3\sqrt{73}$.

58. The sides of a 30-60-right triangle are n, 2n, and $n\sqrt{3}$ respectively.

59. The hypotenuse of an isosceles right triangle is $\sqrt{2}$ times one of the equal arms.

60. If the legs of a right triangle are 6 and 8 units respectively, the hypotenuse is $\sqrt{14}$ units.

61. If the hypotenuse of a right triangle is 17 inches and one arm is 8 inches, the other arm will be 15 units.

62. A wire that reaches from the top of a telephone pole 43 feet high to a point on the ground 15 feet from the foot of the pole will be slightly less than 55 feet long.

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63. A gate 4.5 feet high and 12 feet long is to be braced from the lower right corner to the upper left. A board 13 feet long will be slightly too long.

64. The flag pole on the school grounds is approximately 70 feet high.

65. If a tree 90 feet tall breaks off 30 feet from the ground and the top strikes the ground, the top forms a 30 degree angle with the ground.

66. If the acute angles of a right triangle are 30 degrees and 60 degrees, the perpendicular to the hypotenuse divides the hypotenuse in the ratio 1 to 3.

67. The mean proportional between two unequal line-segments is greater than one half their sum.

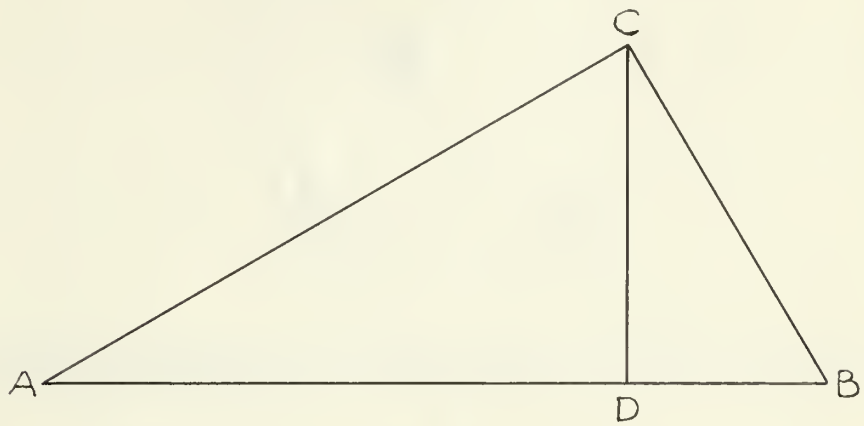
68. The proof of the following theorem makes use of congruent triangles: Two triangles are similar if an angle of one equals an angle of the other and the including sides are proportional.

69. The study of similar polygons started with a new definition and a new postulate.

70. Triangle ABC is similar to triangle A'B'C' if angle A = angle A', AB = k·A'B', and AC = k·A'C'.

71-78 Read the following:

Prove that the product of the two arms of a right triangle is equal to the product of the hypotenuse and the altitude upon the hypotenuse.




Given: right triangle ABC with altitude CD to hypotenuse AB.

Prove $AC \cdot BC = AB \cdot CD$

Circle the numbers of the following statements if they are necessary in the proof of the above. Cross out the numbers of the statements not necessary for the proof.

71. Angle CAB = angle CAD.

72. Right triangle ABC is similar to right triangle ADC.

73. Right triangle ADC is similar to right triangle BCD.

74. $\frac{AB}{BC} = \frac{BC}{DB}$

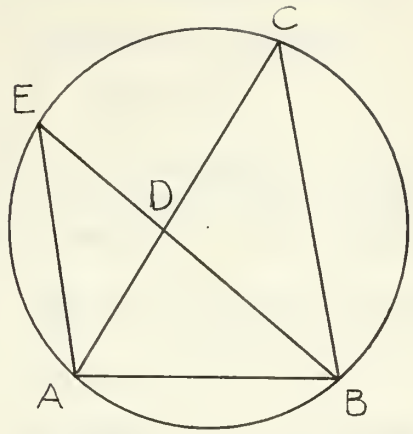
75. $\frac{AB}{AC} = \frac{AC}{AD}$

76. $\frac{BC}{CD} = \frac{AB}{AC}$

77. $\frac{AD}{CD} = \frac{CD}{DB}$

78. $AC \cdot BC = AB \cdot CD$

79-85 Read the following:




Given: Inscribed triangle ABC in which chord BE bisects angle B and chord AE is drawn.

Prove that triangle ABE is similar to triangle BDC.

Circle the numbers of the statements necessary as reasons in the proof. Cross out the numbers of any statements not necessary as reasons.

79. The bisector of an interior angle of a triangle divides the opposite side into segments proportional to the adjacent sides.

80. The bisector of an angle divides it into two equal parts.

81. Angles inscribed in the same arc of a circle are equal.

82. Two triangles are similar if two angles of one are equal to two angles of the other.

83. Two triangles are similar if an angle of one equals an angle of the other and the including sides are proportional.

84. In any proportion the product of the means equals the product of the extremes.

85. Two right triangles are similar if an acute angle of one equals an acute angle of the other.

# GEOMETRY EXAMINATION

## SIMILAR TRIANGLES

## SECTION B

Directions: Do any necessary computation in the left and lower margins or on the back of the page. Write answers on the lines provided at the right of each exercise.

1. Given: Triangle ABC is similar to triangle A'B'C'; AB = 10 inches, AC = 12 inches, and A'C' = 4 inches; A'B' = _____. 1)_____

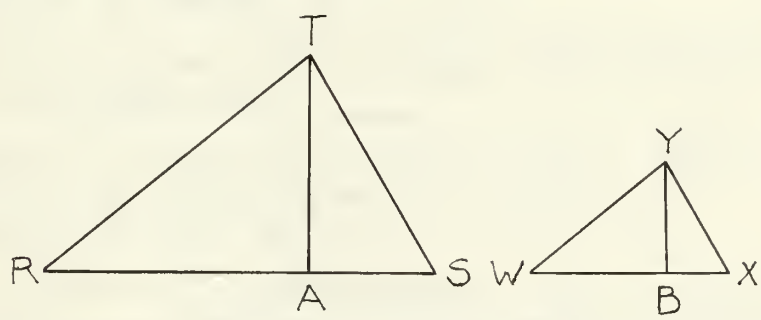
2. The sides of a triangle are as follows; RT = 6 inches, RS = 7 inches, and ST = 8 inches. In similar triangle R'S'T', R'T' equals 12 inches; R'S' = _____. 2)_____

3. If one inch represents 300 feet, _____ inches will represent 750 feet. 3)_____

4. If 3/4 of an inch represents ten feet, _____ feet will be represented by $2\frac{1}{4}$ inches. 4)_____

5. Triangle ABC is congruent to triangle A'B'C' and $\frac{AB}{A'B'} = k$; k = _____. 5)_____

6.




Triangle RST is similar to triangle WXY. If RT = 12 feet and WY = 8 feet, then the ratio of the altitudes AT and BY is _____. 6)_____

7. A boy 5 feet tall casts a shadow 4 feet long at the same time that a nearby telephone pole 35 feet high casts a shadow _____ feet long. 7)_____

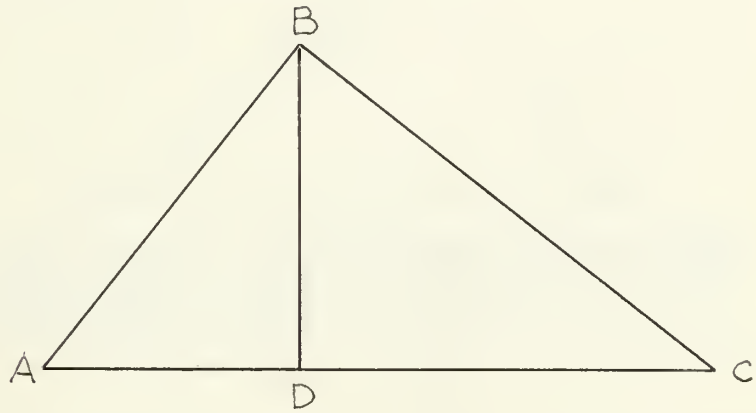
8. A flagpole casts a shadow 84 feet long. At the same time a nearby 12 foot post casts a shadow 21 feet long. The flagpole is _____ feet high. 8)_____

9. AB and CD are two chords of a circle intersecting at E. If AE = 6 inches, EB = 8 inches, and CE = 12 inches, CD = _____. 9)_____

10. BA is a tangent to a circle and BC is a secant, intersecting the circle at D. If BD = 6 inches and CD = 8 inches, AB = _____. 10)_____

11. CBA and CDE are secants to a circle, intersecting the circle at B and D, respectively. If AC = 2 BC, EC = 16 feet, and DC = 9 feet, AC = _____. 11)_____

12, 13.




Given: right triangle ABC with right angle at B and DB perpendicular to AC. If DB = 20 inches and AD = 16 inches, DC = _____. 12)_____

If in the above right triangle, AB = CB and if AC = 14 inches, AB = _____. 13)_____

14. (Answer in simplified radical form.) If the altitude of an equilateral triangle is 6 inches, the side is _____. 14)_____

15. In a circle of 4 inch radius, a chord 6 inches long is _____ inches from the center of the circle. 15)_____

16. If the arms of a right triangle are 5 inches and 13 inches in length, the hypotenuse is _____ inches. 16)_____

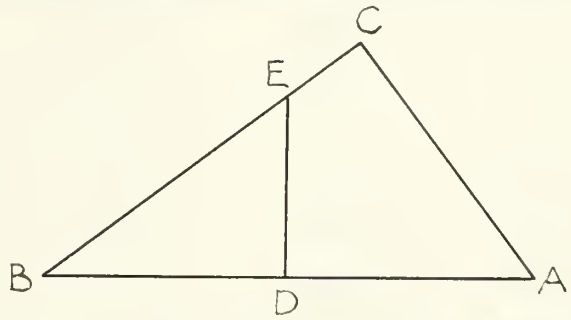
17. A point P is 4 inches from the center of a circle with a 6 inch radius. Chord BC passes through P so that BP = 5 inches. The length of the chord BC = _____. 17)_____

18. A fan window surmounts a door 6 feet wide and the center of the arc of the window is 2 feet above the middle of the door. The radius of the arc of the window is _____. 18)_____

19. The arms BC and AC of right triangle ABC are 8 and 6, respectively. Hypotenuse BA = _____. 19)_____

20. In the figure for exercise 19, CD, the altitude to the hypotenuse equals _____. 20)_____

21.




D is the midpoint of hypotenuse BA of right triangle BCA and DE is perpendicular to BA. If BC = 8 and AC = 6, DE = _____. 21)_____

Geometry Examination Key

Similar Triangles

Section A

| Test Item | Response | Test Item | Response | Test Item | Response | Test Item | Response | Test Item | Response |
|---|---|---|---|---|---|---|---|---|---|
| 1 | T | 18 | F | 35 | T | 52 | T | 69 | F |
| 2 | T | 19 | T | 36 | F | 53 | F | 70 | T |
| 3 | T | 20 | T | 37 | T | 54 | T | 71 | T |
| 4 | T | 21 | T | 38 | F | 55 | T | 72 | T |
| 5 | T | 22 | F | 39 | T | 56 | F | 73 | F |
| 6 | T | 23 | T | 40 | F | 57 | T | 74 | F |
| 7 | F | 24 | T | 41 | T | 58 | F | 75 | F |
| 8 | T | 25 | T | 42 | F | 59 | T | 76 | T |
| 9 | F | 26 | T | 43 | T | 60 | F | 77 | F |
| 10 | F | 27 | F | 44 | T | 61 | F | 78 | T |
| *11 | T | 28 | T | 45 | T | 62 | F | 79 | F |
| 12 | F | 29 | T | 46 | T | 63 | T | 80 | T |
| 13 | T | 30 | T | 47 | T | 64 | T | 81 | T |
| 14 | F | 31 | F | 48 | F | 65 | T | 82 | T |
| 15 | T | 32 | F | 49 | T | 66 | T | 83 | F |
| 16 | T | 33 | T | 50 | T | 67 | F | 84 | F |
| 17 | F | 34 | F | 51 | T | 68 | F | 85 | F |

*By law the American flag is a rectangle 10 by 19 or a similar rectangle.

## Comment on the Key to Section A

Pupils' method of marking the test.-- The directions on the pupils' examinations were to circle the number of the test item if the item were true and to cross out the number if the item were false. The teacher's copy of the key was prepared with similar marks and was so arranged that it was folded in sections which coincided with the left margin of the pupil's paper. However, for facility in typing, the arrangement on the preceding page was used.

Geometry Examination Key

Similar Triangles

Section B

1. 3 1/3 inches
2. 14 inches and 16 inches
3. $2\frac{1}{2}$ inches
4. 30 feet
5. 1
6. $1\frac{1}{2}$ or $\frac{3}{2}$ or 3:2
7. 28 feet
8. 48 feet
9. 16 inches
10. $2\sqrt{21}$ or $\sqrt{84}$ or 9.165 or 9.2 inches
11. $\sqrt{72}$ feet or $2\sqrt{36}$ feet
12. 25 inches
13. $7\sqrt{2}$
14. $4\sqrt{3}$
15. $\sqrt{7}$ inches
16. $\sqrt{194}$ inches
17. 9 inches
18. $3\frac{1}{4}$ feet
19. 10
20. 4.8 ($\sqrt{25.04}$ not a good answer, but accepted)
21. 3 3/4

Other Methods of Evaluating the Pupil's Work

Teacher's observation.-- In addition to the objective tests, there are other methods by which a student's work in the unit will be evaluated. These questions give a clue to his grasp of the subject.

1. How does he attack a theorem or an exercise? Does he have a systematic plan or does he follow a desperate hit-or-miss scheme?
2. Does he recall the proof of a theorem through understanding or does he depend on memorizing?
3. Is he a help or a hindrance in group work?
4. Can he concentrate on new work that is being developed in class and does he offer constructive suggestions?
5. Is his previous information organized so that it is available for use in new situations?
6. Can he give a good demonstration of theorems or exercises?
7. Can he follow the proof of a fellow student and detect errors or omissions?
8. Can he think on his feet and correct his own errors?
9. How good is he at drawing the figure and detecting the hypothesis and conclusion in a new exercise.

10. Does he confine his efforts to easy work or can he successfully attempt difficult problems?
11. Are his questions intelligent or do they show a feeble grasp of the situation?

The above questions may be answered through teacher observation of the pupil. His mark on the objective test may be conditioned by reading ability, by incipient illness, or some emotional factor, while his work over a period of time may disclose real ability in the subject.

Therefore, the final evaluation of the pupil's progress in the unit will be based upon his standing on the objective test and upon the teacher's judgment as to how well his class work, group work, and individual work have disclosed the qualifications indicated in the foregoing questions.

## CHAPTER III

## Teaching the First Unit

The teacher's log.-- This account of the teaching of the unit was prepared from the log which had been kept by the teacher during the progress of the unit. Entries were jotted down during the period when time permitted, or at the end of the period. Since school activities were conducted on an informal plan and pupils were free to drop in to consult teachers between periods, and especially since pupils for whom the unit had been organized were in no hurry to leave the classroom, these entries could not be as complete as the teacher would have wished. They were in the form of notes and concerned the highlights of the work of the period.

Introduction of the unit.-- On Monday, May thirteen, nineteen hundred forty-six, the last twenty minutes of the period were given to the introduction to the unit. The first twenty minutes of the period had been devoted to an arithmetic test given by the head of the mathematics department. When the test was completed, the teacher referred to the blue-print displayed on the bulletin board and the pupils explained the purposes of blue-

prints. They described some which they had seen. Of all the values of a blue-print, stress was laid on the idea that it was a representation on a diminished scale of an original figure, or that in such cases as small parts of machines, it was a representation on an enlarged scale.

Preliminary diagnostic test.-- In most cases of teaching by the unit method, the next step would be the administration of the preliminary test to determine how much previous information the pupils possessed on the topic to be presented. Since the mimeographed copies of the test were not ready, pre-tests were not given.

Pupil study guides.-- The usual procedure after the preliminary test has been given is to distribute copies of the pupil's study guides. There had not been time to have them mimeographed, as explained in Chapter One, so a typewritten copy had been placed on one bulletin board and the carbon copy on the other. Also posted on the bulletin board were copies of the list of optional related activities and a list of pupils' reference books. There was time during the introductory period to explain the code system used and to read and discuss enough of the study guide so that the pupils knew how to plan their approach to the unit. They decided to start the work in groups and to arrange their own groups.

Second day of the unit.-- On Tuesday the members of

the first class, referred to hereafter as Group I, arrived in a great hurry and before the teacher had returned from five-minutes corridor supervision, they had arranged the desks in groups of two's, three's, or four's, had referred to notes taken previously from the study guides, had procured reference books, and were hard at work. The teacher noticed, on entering, that almost all the members of all groups were talking at once, but said nothing to the class, preferring to await further developments. The chief questions referred to the teacher centered about the choice of exercises. Pupils wanted to be reassured that they could start with the work that appealed to them individually. Three boys started with a scale drawing of an army hut and soon asked permission to work at the blackboard rather than on paper. Two others chose a scale drawing of a trestle bridge in which the distance from the foot of a ravine to the floor of the bridge was to be found. The teacher suggested that they use large drawing paper so that the solution could be compared later with the solution arrived at by a different method. Eventually, both methods of solution became part of the bulletin-board display. Other pupils began the study of the book definition of similar polygons or turned to the theorems about similarity of triangles. Some groups read the theorems and started exercises, the solutions of which depended upon the proportions

involving corresponding sides. Thus, at the first real period of study, some pupils had worked beyond the first predetermined instruction point. When the teacher joined each group in response to the members' requests for help, she first of all suggested a quieter tone of voice and spoke quietly herself. It was not long before the other groups noticed what was going on, and each group took care of its own overenthusiastic members. Naturally, with eight or so groups of alert, interested, and at first somewhat excited boys and girls, there was a bustle and stir, but the concentration in each group was so intense that very rarely were members of one group distracted by the activities of the other groups. The teacher did not have a breathing spell all period. As soon as she left one group, two or three others requested her presence. No group sat around idly waiting for her but continued the discussion among its own members while waiting for her. Also, as soon as the teacher had corroborated a pupil's point of view, had referred a group to a source of information, had suggested that they had overlooked a necessary definition, or had admired a finished piece of work, the group detained her no longer. They took pride in accomplishing as much as possible on their own initiative.

The second class, Group II, four periods later in the day, entered the room and sat in regularly assigned seats

and waited for the teacher to begin the lesson. When she expressed surprise and reminded them that they were to have organized the work in their own way, they went into action, but in a quieter, calmer way than had the younger pupils.

The third day.-- Before school on Wednesday, a boy in Group I arrived with two blue-prints of marine engines which had been lent by his father, a marine engineer. He removed material from the second bulletin board at the front of the room and replaced it with the blue prints.

When Group I reported to class that day, they were in such haste to arrange the seats that the room could not be described as quiet, but it was an improvement over the preceding day. The preliminary conversation of the pupils sounded much more controlled because the teacher had privately requested four boys who had used the loudest tone of voice on Tuesday to refrain entirely from speaking on Wednesday until ten minutes of the period had elapsed. It was amusing to see them get to work with one finger occasionally pressed to their lips and communicating by nods, shakes of the head, or by pointing to sections of their books. When the ten minutes were up, they conversed quietly. When a group became too excited over a difference of opinion, the teacher joined the group and reminded the members that none of them could listen if all were talking at once. Some groups still worked on scale

drawings, some studied the new bulletin-board material, others swapped reference books, and others submitted completed work and asked if it were suitable for program material.

As the teacher had responded to requests for help, she had suggested that certain material deserved a place on the program. Without setting an actual date for this program, she started listing on the board the title of the material and the name of the pupil who volunteered it. Before the end of the period, pupils were bringing her completed work and asking if it could be placed on the program.

At the very end of the period, the teacher secured the attention of all pupils and commended them for the rapid progress they seemed to be making in geometry and then raised the question of good manners in conversation. The group seemed abashed at realizing that, as the period progressed, their eagerness led them to grow careless in regard to certain conventions, but they agreed that they could remedy the situation themselves. One of the four worst offenders of the previous day evoked a good laugh when he suggested that the regulations in force that day at the beginning of the period be reversed on Thursday and that his group be the only one allowed to talk for the first ten minutes.

Group II progressed in a manner much the same as had Group I, but again, in a calmer, cooler fashion. Absentees, who returned knowing nothing of the unit, were absorbed by groups and initiated by the group members without any necessary intervention on the part of the teacher. The three boys working on the scale drawing of the army hut showed the most interest in the blue-prints of the marine engines. Questions were asked about the program for the first period class, and Group II started to list activities for its own program.

The fourth day.-- On the fourth day the first class began and continued to work quietly without any reminders from the teacher. Pupils of a particular group took care of any one of their group who became overly enthusiastic. They were still solving exercises for the program and were proving theorems regarding similar triangles. Some of the slower pupils chose the most difficult theorem and were advised to become more familiar with the definitions and the preceding theorem before spending more time on the one which was causing them so much trouble. Pupils who had been attempting problems involving photography were pleased when they saw that from the classroom windows they had full view of the graduating class which was being photographed. They watched the proceedings for a while and estimated the height of the stands plus the height of the tallest pupil in the center of the top row, the

distance of that row from the camera, and the focal length of the camera, then went to the board and tried to compute the approximate height of the same pupil as he would appear in the photograph. It was a difficult problem for them as they had not studied the theorem necessary for its solution, and it was still unsolved at the end of the period. One dreamy lad showed more animation than at any time during the year and requested permission to take home the reference book which had first interested the group in such problems. At the end of the period the pupils decided that they had a full program prepared, so planned to have the program on Friday. Pupils obtained "change-of-room" permits and spent their study periods on Thursday putting the necessary figures on the blackboards. Others obtained written permission to enter the building early on Friday so as to have everything in readiness for their share of the program.

Group II spent the time in proving theorems to one another, with certain groups joining forces when they found they were working on the same theorems. By this time some pupils had noticed that the first of the theorems on similar triangles was stated differently in some of the texts, and that the first theorem in their textbook was the corollary to the first theorem in a reference book. One pupil, who had been working ahead of the others,

joined the discussion and showed to the satisfaction of all that there was no contradiction involved in the different arrangements. The group working on the army hut completed a beautifully drawn scale drawing, added an elaborate drawing of a compass, and labeled it Belliveau, Poch, Stevens, Architectural Designers. Much of the format had been acquired from their study of the marine engine prints. The teacher had tried to discourage them from spending so much time on it, but they assured her that they would see to it that they did not lose out on the core material.

Aerial photographs.-- On Thursday, after school, a former pupil, then in the Army Air Corps, dropped in for a visit. When he saw the work which the classes were doing, he offered to donate a set of air photographs taken by students at Fort Belvoir, Virginia. These were sent by him on the following Monday and were given to a committee of pupils, headed by the boy in the Civil Air Patrol. The photographs were arranged in sequence, mounted on the bulletin board and explained to the class the following Wednesday.

The first program.-- On Friday, one girl in Group I was waiting to enter the school at 8 a.m. so as to draw the figure for her exercise. She was a new girl at the school who had been having difficulty with geometry, and

she had been in the habit of waiting listlessly for help from the teacher. She and a friend had chosen an exercise involving similar triangles.

When the bell rang at the beginning of the period, the pupils who were still at the board putting the finishing touches to their work took their places in their groups, and the girl who had entered early explained the first exercise. A boy in the class asked her how she knew the two triangles in the figure were similar. She referred him to theorem 55 of the textbook. The eighth boy on the program spoke up and said, "That is the theorem that I am going to prove. Do you want me to prove it now?" The pupils who had signed for the first two theorems pointed out that they would have to prove their theorems before he could logically prove his. Thus, although the program had been jotted down in the order in which the pupils had volunteered to do exercises and theorems, the logical order was arranged by the pupils themselves, once the program started. Other teachers might prefer to rearrange the program beforehand and present it to the pupils in logical order. The teacher of this unit was curious to see what would happen so had said nothing to the class about the sequence of theorems and exercises. It was with no small sense of satisfaction that she observed the pupils straighten out the situation for themselves.

In addition to the three theorems which were proved, the large scale drawing of the trestle bridge was explained by one of the boys who had drawn it. The computation had been checked by several boys in the group so that this number on the program took very little time. The drawing was part of the display on the bulletin board above the blackboards. The boys found the result to be approximately twenty-two feet. Later in the unit the same boys solved the problem by means of the 30-60-right triangle involved and found the solution to be twenty-five feet. They took down the drawing from the bulletin board to try to approximate more closely the new result.

There were three exercises involving circles proved next, two from reference books and one from the text.

Group II.-- Only two pupils in Group II had appeared before school or during a study period on Friday, and the teacher was becoming apprehensive about the success of their program. At the beginning of the period, however, they made up for lost time. The three pupils who were to prove the theorems asked permission to use the figures left on the board by Group I. The pupils with exercises started to put on quickly drawn free-hand figures, while the owners of the drawing of the army hut stood guard to see that it was not molested.

Order of events.-- The program for Group II did not range over so wide a field as had that of Group I. The

first number was the first exercise in the text and involved a construction, the second was a numerical exercise, the next two were from reference books and of comparative difficulty with those chosen by Group I, the next was from the text and concerned intersecting chords of a circle. Then came the proofs of the theorems and the description of the army hut. The proofs of the exercises were accepted with the comment that they depended upon the proofs of the theorems used, and the last two theorems were proved very well by two of the best pupils in the class. The girl who attempted the first of the theorems realized that she had not done it well and asked for more time to study. This class spent less time on discussing the proofs that were presented. One pupil asked about shadow measurement, and the class was looking this up in the text and in the reference books when the period ended.

Summary of the work of the first week.-- The classes did not wait for the bell to ring to begin class. They hurried in, sometimes four minutes before the final five-minute passing period and started work as soon as they arrived. They were noisy at the beginning of the period chiefly because of moving furniture and of eagerness to get to work. As the week progressed, a quiet tone of voice replaced the excited chattering. There was so much interest in exercises that some pupils tended to accept

the new theorems without proving them, but during the Friday program they became more aware of the importance of the proofs and decided that for the rest of the unit they would learn to prove the new theorems in sequence. Items 1 to 14 in the delimitation had been studied by many of the pupils and had been included in the oral summaries in the program. The pupils also expressed the desire to continue with this method of study. There had been no home work assignments given, but pupils were accomplishing more than if there had been.

Lack of equipment.-- There was a very evident need of more reference books, and the teacher checked with the office to see what was being done with regard to the list of reference books which had been approved by the superintendent of schools. The books began to arrive late in June and early the next September, too late for the first teaching of the unit, but in time to be of value for the final review of the year's work and for the next year's classes. The necessity of a large bulletin board to display the work, which, for lack of time could not appear on the programs, was apparent. The written work done as part of the unit was exceptionally neat. Individual pupils were improving their own best previous written work and were disappointed when there was no suitable place to display all of it.

Highlights of the second week, May 20.-- The arithmetic tests took twenty minutes of the period on

Monday. The previous week most of the pupils had gone beyond the first previously determined instruction points, items 4, 8, 10, and 13 in the pupils' study guide, so the teacher had changed her plan. Some of the important points had been stressed for the class as a whole during the program or pooling-of-experience phase on Friday. Others had been presented to small groups in answer to their questions.

Each class held a class conference to clear up the method of proving that if two chords intersect within a circle the product of the segments of one is equal to the product of the segments of the other. At this time the father of a girl in Group II sent in the parts of a broken gear. The illustrated solution of this problem from a clipping from Life magazine was in the files, and a pupil was sent to find it. The pupils in each group took measurements from the largest segment of the broken gear and, using item 18, worked out the length of the radius of the gear. Since the father had thoughtfully sent in all the pieces, the practical members of the class were able to fit the pieces together and, by direct measurement, check the first result. This brought up a discussion as how best to measure it directly and furnished a good review of the approximate nature of measurement. The parts of the gear, the magazine clipping, proof of the theorem,

and neat reports of the computation were arranged by a committee of pupils as a table display.

Measurement of the flag pole on the school grounds was planned and conducted, two groups from each class being allowed out-of-doors at the same time. This was an optional related activity. Pupils who had been trying to avoid the theorems or corollaries were seen referring to books again, as the written and oral reports were supposed to include the proofs as well as the numerical result.

The pupils who had had trouble with the photography problems had been straightened out after the teacher had suggested in Group I that they try proving exercise 10 on page 330 of their text book. Later, one of them brought a reference book to her and triumphantly pointed out that that particular exercise in their book was a corollary in the reference book, only to be told by a pupil who overheard the discussion that it was also Theorem 57 on page 332 of the text book.

"Why didn't you tell me that it was the theorem instead of the exercise?" the first pupil asked the teacher. The other pupil showed page 332 to the first and tapped the proof with his pencil, whereupon the first one laughed and said, "Oh! Well, I certainly worked on that one!" In Group II the pupils who had attempted the photography problem could not solve it after repeated attempts. After the teacher had taken them to the board

and explained it step by step, they were able to work out all such problems but still were not too well able to prove the underlying theorem.

The best pupils in each class were the ones who most enjoyed proving the theorems and solving the exercises dealing with abstractions, although they did not totally ignore the practical problems. The poorest pupils chose first the exercises with the most interesting applications and were content with a rule-of-thumb method of solution.

Third week: Memorial Day Week.-- Item 24, which necessitated the solution of quadratic equations, had been studied by many of the quickest pupils the second week. However, when an equation could not be solved quickly by inspection, the teacher had suggested that they study page 337 in the text or leave the equation unsolved until the class lesson on solution by completing the square. On Monday after the arithmetic test, this previously determined instruction point was taken up. The pupils who had mastered it by themselves were excused and worked quietly on advanced items. As soon as pupils were able to take over the explanation and solutions, the teacher excused them from the larger group, and before the end of the period, there was only a small group working with the teacher at the blackboard. The rest of the class was looking up such exercises in reference books and explaining them to one another in their original groups.

There were three working days for Group I in the third week and two for Group II. The Memorial Day assembly took place during the geometry period of Group II, and there was no school on the next two days. The work had been interrupted also by pupils in the musical organizations being excused for rehearsals.

Fourth week: June 3.-- The arithmetic test was omitted on Monday, June 3, and an instruction period on the theorems about the altitude to the hypotenuse of any right triangle was held. This had not been provided for in the original form of the pupils' study guide, but experience with the pupils who were most advanced in the unit indicated that these theorems and exercises were difficult for pupils to understand without assistance. They caused more trouble than the theorem about the secant and tangent drawn to a circle from an external point, probably because of the three right triangles involved. When such an instruction period was held, the attention and concentration of the pupils were high. They had learned that such periods were held only when the work was of such a nature that they would probably need help or because it involved an unusually interesting or practical application.

The teacher recommended that pupils who were still working on items 22 and 24 leave them and join in the work on right triangles. She was satisfied if they could solve the easiest exercises involving secants and tangents.

The fastest working pupils were finding that item 35 offered challenging work, and they were visiting back and forth among groups to compare numerical results on exercises that interested them the most. In some cases they kept a record of the different methods of proving exercises and congratulated the pupil who discovered the shortest correct method.

Pupil assistance.-- Pupils had been allowed to form their own groups. In most cases, the best pupils had grouped together, but in some cases, through friendship perhaps, a slower working pupil was a member of a fast group. In such cases, the other members of the group made themselves responsible for helping that pupil, and, in class time, could be observed explaining material or having the slower pupil prove theorems to them. At other times, in good-natured exasperation, they would pick out easy exercises for such a pupil or refer him to the teacher or to another group so that he would not interrupt their concentration on a difficult exercise. Two boys in two groups of all boys of the rough-and-ready type had developed into real group leaders. They were boys who had been content to get by with a mark of L or C so that they would not be ineligible for athletics. In making themselves responsible for their fellow athletes in their groups, they found that geometry did make sense after all, and that they could do excellent work. Their pleased surprise

at their own accomplishments, the masterful way in which they kept members of their group at work, and the evident relish with which they took part in the programs or bulletin displays, were a decided contrast to their previous attitude twoard the subject.

Bulletin boards again.-- By this time the teacher had reserved the first and last sections of the side blackboard for bulletin board space. Pupils attached material directly to the blackboard by Scotch tape. The drawback to this arrangement was that former material could not be rearranged when newer material was added, so the displays were not as logically arrangednor as pleasing to the eye as they otherwise would have been.

Optional related activities.-- The first evidence that optional related activities were being consulted was the present of the gear, which was mentioned in the fourth item. Instead of the material listed in the fifth item, some pupils brought in scale drawings of plans for boats or for furniture which they had made in mechanical drawing or in shop. The teacher regretted pupil choice of items six and nine, references to photography, and planned to restrict them in the future to the latter part of the unit. Item 8, the flagpole reference, was so popular that it was more like a core activity. Items 10 to 15 were not chosen by anyone. One girl chose number 16 and prepared a four-page report on Pythagoras. There was not time to

give the report in class, so the teacher suggested a bulletin-board display which could be consulted at the convenience of the pupils. The girl found a picture of Pythagoras in the files and mounted it on green drawing paper. She collected from her classmates well-written proofs of the Pythagorean theorem and exercises solved by means of the theorem. With these and her written report she and two friends arranged another attractive and instructive display which was consulted by members of both classes as well as by members of the teacher's three eleventh-grade college-mathematics classes. Item 18, President Garfield's proof of the Pythagorean theorem, had been chosen by a group of boys, but the teacher was unaware of it until a year later, when, as members of her eleventh-grade course, two of the boys reproduced the proof and said that they had become interested in it while working on the unit the previous year.

End of class work.-- The fourth week would end the tentative three-and-a-half week time allottment, since the third week had been only a partial week. Therefore, on Thursday the teacher sounded out the class on the date for the tests on the unit. Most of the pupils would have liked one week longer before the tests were given but were reminded by the teacher that there was no rule against home study and that they had Thursday, Friday, and the week-end in case they needed to do extra studying on

item 37, the sample examinations, or to review the work of the unit. She agreed that on the next Monday she would answer any questions that remained unanswered and help the class with any exercises that they could not do in the sample tests. This would be the last pooling-of-experiences period. Section A of the final examination would be given on Tuesday, June 11, and Section B on the following day. The commercial department had mimeographed the 85 items of Section A. The test required six pages, stapled together, for each of the 53 pupils. Section B was written on the blackboard in the same form as would have been used had there been time to mimeograph it.

## CHAPTER IV

## Evaluation of the Unit

Section A of the final examination.-- To evaluate the unit on similar triangles a separate study was made of each part of the final examination before the study of the total scores was made. The facts for the first study were obtained from Table 17, Appendix A. Table 5 shows the method used to find the arithmetic mean and standard deviation for fifty-three pupils on Section A of the final examination. To make them easier to follow, all such tables will be placed at the top of the following page, thus making it unnecessary to turn a page while reading the table.

Table 5. Arithmetic Mean and Standard Deviation Scores of Fifty-Three Geometry Pupils on Section A of the Similar Triangle Examination in a Grouped Frequency Distribution.

| Class-Interval, 5 Limits | Mid-point | Frequency f | Deviation d | Frequency Deviation fd | Product of Columns (4 and 5) $fd^2$ |
|---|---|---|---|---|---|
| (1) | (2) | (3) | (4) | (5) | (6) |
| 71-75 | 73 | 2 | | | |
| 66-70 | 68 | 4 | +3 | +12 | 36 |
| 61-65 | 63 | 8 | +2 | +16 | 32 |
| 56-60 | 58 | 11 | +1 | +11 | 11 |
| 51-55 | 53 | 10 | 0 | 0 | 0 |
| 46-50 | 48 | 9 | -1 | -9 | 9 |
| 41-45 | 43 | 4 | -2 | -8 | 16 |
| 36-40 | 38 | 1 | -3 | -3 | 9 |
| 31-35 | 33 | 3 | -4 | -12 | 48 |
| 26-30 | 28 | 0 | -5 | 0 | 0 |
| 21-25 | 23 | 1 | -6 | -6 | 36 |
| Totals | | 53 | | +1 | 197 |

**Mean**

$$\frac{\Sigma fd}{N} = \frac{+1}{53} = +.019 = c$$

$$\frac{\Sigma fd}{N}(s) = 5(.019) = .095$$

$$M = 53 + .095 = 53.095 \text{ or } 53.10$$

**Standard Deviation**

$$S.D. = s\sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2}$$

$$\Sigma fd = +1 \qquad S.D. = 5\sqrt{3.7169 - .0006}$$

$$c = \frac{\Sigma fd}{N} = \frac{+1}{53} = +.019 \qquad = 5\sqrt{3.7165}$$

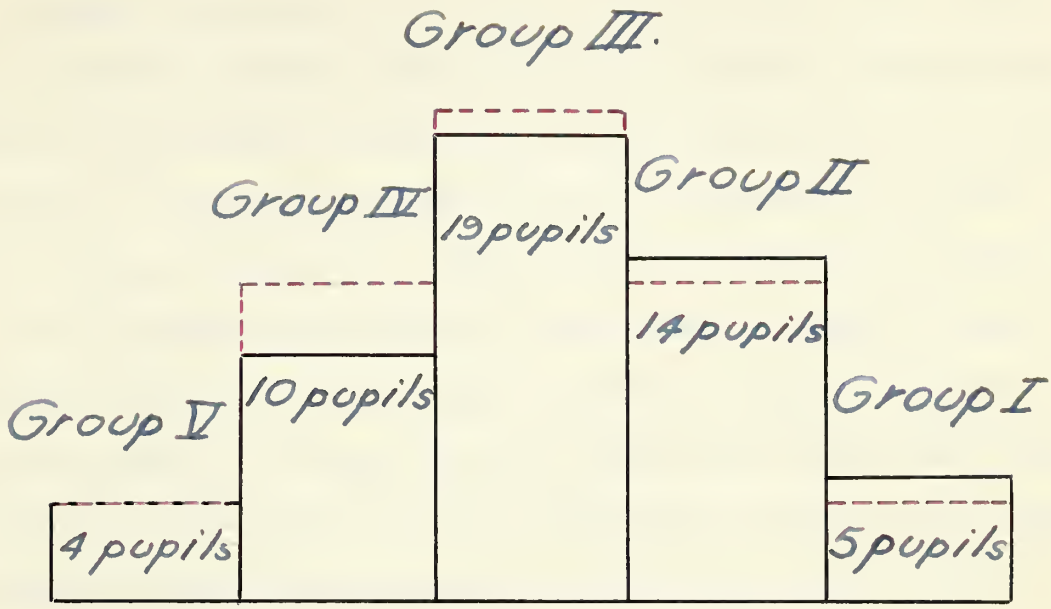
$$c^2 = (+.019)^2 \qquad = 5(1.928)$$

$$= .0004 \qquad S.D. = 9.640$$

$$\Sigma fd^2 = 197$$

$$\frac{\Sigma fd^2}{N} = \frac{197}{53}$$

$$= 3.717$$




Actual distribution of pupil growth with the normal distribution in red.

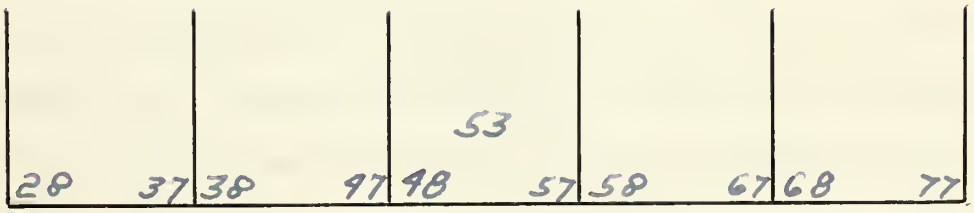



Figure 1. Relative growth scale - Section A of the similar triangle test.

Table 5 shows the arithmetic mean to be 53.10 and the standard deviation to be 9.64. These results were used for the relative growth scale (Figure 1) and for the histogram which were drawn to show the actual distribution of pupil growth. The normal distribution is represented by the red lines. (Figure 1). The results of Section A of the final examination on similar triangles as shown by the actual distribution of pupil growth in figure 1 showed fewer pupils in groups IV and V than the teacher had expected. One pupil had a score of 25, which fell outside the lowest point in the relative growth scale, but it was included in group V in the histogram. This pupil was a boy whose intelligence quotient was 114. As a ninth-grader the previous year he had borrowed eleventh-and twelfth-grade mathematics books and had given the impression that he was a prodigy in mathematics. The tenth-grade teacher, after a short time, had been astonished at his store of misinformation. The boy found it difficult to adjust his careless thinking to the rigorous demands of deductive reasoning but was keen enough to pick up sufficient information from the recitations of his classmates to just scrape by in the examinations. During the course of the unit the pupils in his group became impatient with him and charged him with wasting their time in pointless discussion. The teacher tried to steer him onto the right track but finally

realized that his aim was to learn the work without any expenditure of energy on his own part. His low score in the examination was the best possible way of convincing him of the necessity for real study.

A distinct advantage of the unit method in geometry is that a pupil must do thorough work during the whole time that the unit is in operation. If he tries to rely on the pooling-of-experience phase, he finds that he cannot learn enough or receive enough practice to do well on the test. The dreamers and the bluffs learn that they must settle down to hard work.

The other three pupils in group V were two girls and a boy who, in the placement tests, had been low in arithmetic reasoning, arighmetic fundamentals, and language. Two of them had been doing failing work in geometry before the unit started.

A significant observation in regard to the actual distribution of pupil growth is that of four pupils who had also been failing in geometry; test results disclosed that two were in group III and two were in group IV. Of the nineteen pupils who had been doing C work previous to the unit, six were in group II in the actual distribution of pupil growth. Thus, ten average or below-average pupils had responded so well to the challenge set by the unit that their scores in Section A of the test exceeded their scores in any written work done up to the beginning of the unit.

The histogram in red shows the normal distribution, where seven per cent of the pupils were placed in each of groups I and V, twenty-four per cent in each of groups II and IV, and thirty-eight per cent in group III. (Figure 1). In all histograms, results were represented to the nearest unit. In groups I and II the actual distribution of pupil growth was slightly higher than the normal distribution. It was lower in group IV and the same in group V.

In appendix A will be found an item analysis of Section A of the test (Table 19), comments on the test, and a list of changes to be made in some of the test items.

Section B of the final examination.-- Table 18 (Appendix A) was used for the report on Section B of the final test. The results are given in Table 6.

Table 6. Arithmetic Mean and Standard Deviation Scores of Fifty-Three Geometry Pupils on Section B of the Similar Triangle Examination in a Grouped Frequency Distribution.

| Class-Interval, 3 Limits | Mid-Point | Frequency f | Deviation d | Frequency Deviation fd | $fd^2$ |
|---|---|---|---|---|---|
| (1) | (2) | (3) | (4) | (5) | (6) |
| 16-18 | 17 | 3 | +3 | +9 | 27 |
| 13-15 | 14 | 5 | +2 | +10 | 20 |
| 10-12 | 11 | 14 | +1 | +14 | 14 |
| 7-9 | 8 | 22 | 0 | 0 | 0 |
| 4-6 | 5 | 8 | -1 | -8 | 8 |
| 1-3 | 2 | 1 | -2 | -2 | 4 |
| Totals | | 53 | | +23 | 73 |

Mean

$\frac{\Sigma fd}{N} = \frac{23}{53} = .434 = c$

$\frac{\Sigma fd}{N} .s = 3(.434)$

$= 1.302$

or 1.30

$M = 8 + 1.30 = 9.30$

$M = 9.30$

$S.D. = s\sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2}$ $S.D. = 3\sqrt{1.377 - .188}$

$\Sigma fd = 23$

$c = \frac{\Sigma fd}{N}$

$= \frac{23}{53} = .4339 = c$

$c^2 = (.4339)^2$

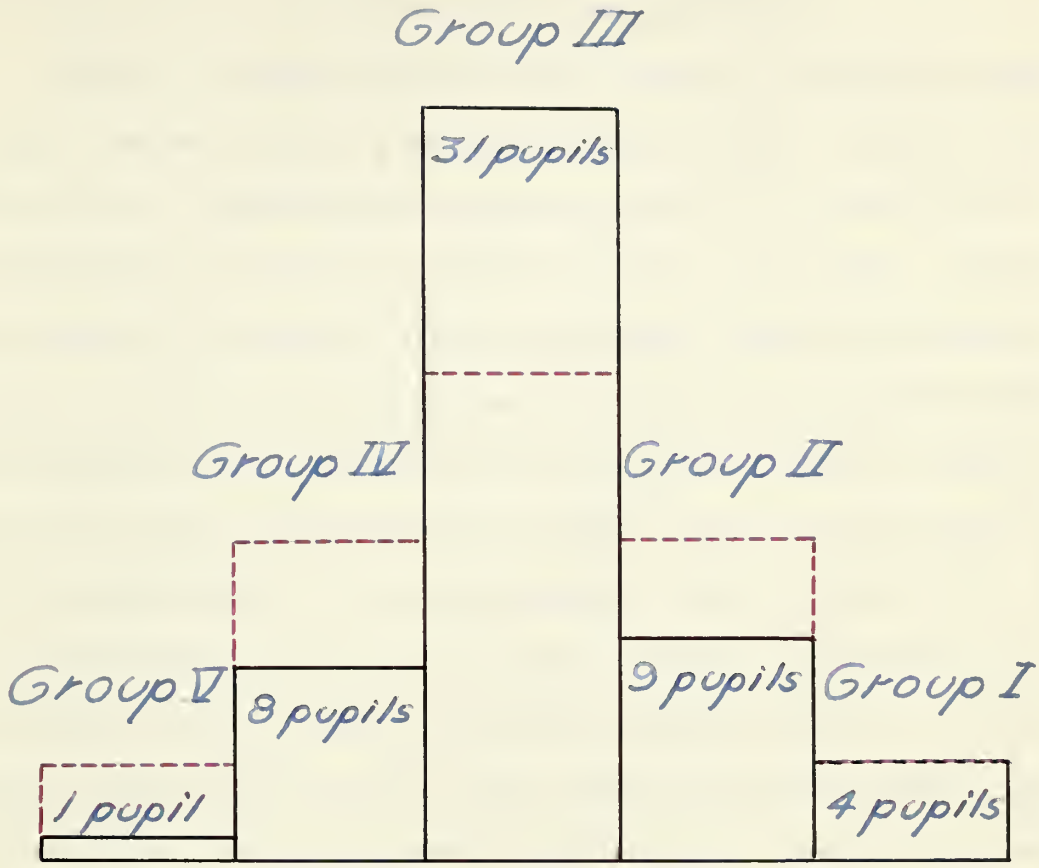
$= .1883$

$\frac{\Sigma fd^2}{N} = \frac{73}{53} = 1.377$

$= 3\sqrt{1.189}$

$= 3(1.09)$

$S.D. = 3.27$




Actual distribution of pupil growth with the normal distribution in red.

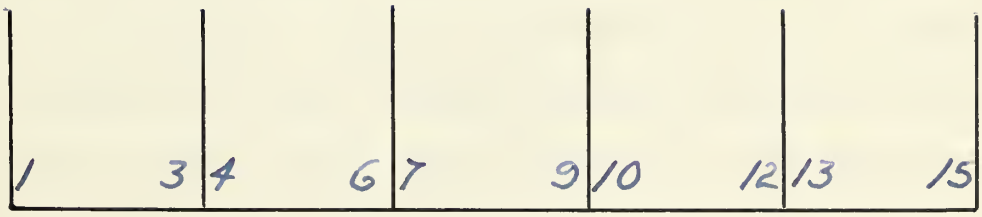



Figure 2. Relative growth scale - Section B of the similar triangle examination.

Table 6 gives the arithmetic mean and the standard deviation for this test as 9.30 and 3.27, respectively.

Figure 2 indicates that the number of pupils in group I was the same for the actual and the normal distributions. The actual distribution of pupil growth in group II was well below the normal distribution for that group. However, the pupils in groups IV and V did much better in Section B than could have been expected from the normal distribution. Four pupils who might have been expected to be in group II in the actual distribution may have lost out on account of the preponderance of right-triangle items in the test. On the other hand, pupils who might have been expected to be in groups IV or V were evidently in groups III or IV which would mean that they did better than might have been expected on the test, including the right-triangle items. A possible interpretation may be that the careful plodders who worked patiently on the routine practice materials were rewarded, whereas brighter pupils who omitted such practice were handicapped in the test. It may also indicate that in the test too much emphasis was placed on the right-triangle phase of the unit. Another year more work will be done on the solution of right triangles during the earlier work of the year so that pupils will not have to take time from the unit to acquire such facility. A further study of Section B of the test will be found in table 20, Appendix A.

Total scores of Sections A and B of final test.-- The arithmetic mean and standard deviation for the total scores of Sections A and B of the final test were 62.90 and 12.20, respectively. The computation was done from table 4 of Chapter I and table 21 (Appendix A). The work is shown in table 7.

The relative growth scale, actual distribution of pupil growth, and normal distribution were depicted in figure 3.

Table 7. Arithmetic Mean and Standard Deviation Scores of Fifty-three Geometry Pupils on Similar Triangle Test, Total Scores of Sections A and B in a Grouped Frequency Distribution.

| Class-Interval, 5 Limits | Mid-point | Fre-quency f | Devia-tion d | Frequency Devia tion fd | $fd^2$ |
|---|---|---|---|---|---|
| (1) | (2) | (3) | (4) | (5) | (6) |
| 86-90 | 88 | 1 | +5 | 5 | 25 |
| 81-85 | 83 | 3 | +4 | 12 | 48 |
| 76-80 | 78 | 5 | +3 | 15 | 45 |
| 71-75 | 73 | 5 | +2 | 10 | 20 |
| 66-70 | 68 | 8 | +1 | 8 | 8 |
| 61-65 | 63 | 10 | 0 | 0 | 0 |
| 56-60 | 58 | 7 | -1 | -7 | 7 |
| 51-55 | 53 | 7 | -2 | -14 | 28 |
| 46-50 | 48 | 2 | -3 | -6 | 18 |
| 41-45 | 43 | 2 | -4 | -8 | 32 |
| 36-40 | 38 | 2 | -5 | -10 | 50 |
| 31-35 | 33 | 1 | -6 | -6 | 36 |
| | | N = 53 | | fd = -1 | $fd^2$=317 |

Assumed mean - 63

$$\frac{\Sigma fd}{N} = \frac{-1}{53} = -.019$$

$$S.D. = S\sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2}$$

Mean

$$\frac{\Sigma fd}{N} = \frac{-1}{53} = -.019 = c$$

$$\frac{\Sigma fd}{N} .5(-.019) = -.095$$

M = Assumed mean + c
63-.10 = 62.90

$\Sigma fd = -1$

$$\frac{\Sigma fd}{N} = \frac{-1.00}{53}$$

c = -.095

$c^2 = (-.095)^2$
= .0090

$\Sigma fd^2 = 317$
= 5.9811

Standard Deviation

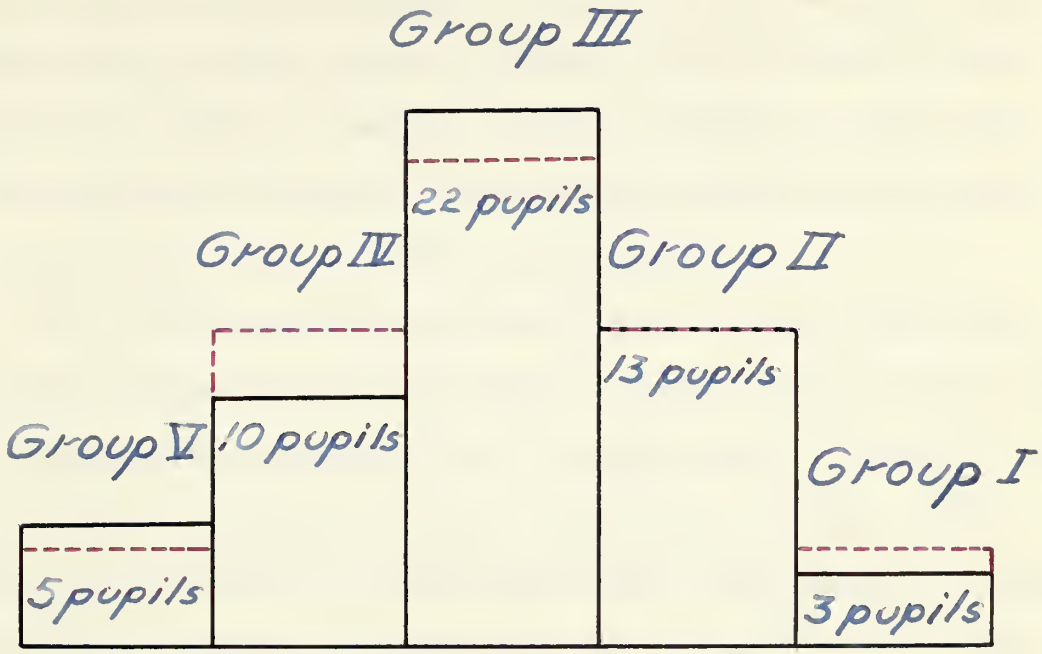
$$\frac{\Sigma fd^2}{N} - c^2 =$$

5.9811 - .0090 =
5.9721

S = 5

S.D. = $5\sqrt{5.9721}$
= 5(2.44)
= 12.20

S.D. = 12.20




Actual distribution of pupil growth with the normal distribution in red.

Figure 3. Relative growth scale - Totals of sections A and B of the similar triangle test.

In all these representations, figures 1, 2, and 3, the relatively small number of pupils involved would indicate that too much emphasis should not be placed on the results as shown. However, it was edifying to note that of the pupils who might have been expected to be in group IV, more were in group III than in group V.

One of the pupils in group V was the boy described in the observations about figure 1. His chief trouble was in personality adjustment and in work habits. Another was the drummer described in the first chapter. His attendance had been poor, and on many days when he was in school, he was too sleepy to concentrate on geometry. He failed the subject for the year and failed again that summer when he studied geometry at a private tutoring school. This had been a case referred to the guidance department early in the year and reported again several times during the year. The intelligence quotient was 97; previous marks in mathematics from grades seven to nine were L, L, L; and grade placements as shown in chapter I were 8.1, 8.3, 7.4, 8.1, 8.8, and 7.4.

The other pupils in group V were the three described as low in previous preparation in the report on Section A of the test.

The three pupils in group I were two girls and a boy, one girl in the 10-3 division and the other two pupils in

10-1. Their intelligence quotients were 142, 145, and 127, respectively. Their previous marks in mathematics from grades seven to nine were A's, with the exception of one B for each of the two girls. All had received A in geometry, and their grade placements as shown in chapter I were almost all 13.0 or 13+. They were happy and well adjusted and were engaged in many extra-curricular activities. They had been enthusiastic workers during the unit, had taken active part in the pooling-of-experience phases, and had exhibited work on the bulletin boards.

The first girl, who had the highest total score, was the girl who had artistic and dramatic talent. She had been the leader of a group of two boys and another girl who had worked together from the beginning of the unit. She was light-hearted and had a merry laugh, and occasionally her group introduced in their discussions subjects outside the realm of geometry, but they quickly returned to work in response to a questioning glance from the teacher. This girl's three fellow workers stood high in group II of the test with scores of 78, 77, and 75. Her score was 86.

The boy, whose score was 85, was a German refugee whose ambition was to become a priest. He had been in the school for two years and was very popular with the other pupils. He had been taught to speak English in his early

childhood in Germany and had no accent or mannerism beyond a rapid, clipped manner of speaking when he became excited. This boy was one of the most enthusiastic about the unit, and his enthusiasm was contagious. The teacher enjoyed talking with the members of his group because they so evidently relished hard work. They proved theorems rapidly and scurried about after reference books to find difficult exercises.

The girl in 11-1 planned to become a college teacher. She had the third highest score of 82. There were three other girls in her group, all of whom had intelligence quotients in the 130's. She had less spontaneity than the two others in group I, perhaps because she was an only child and used to associating with adults more than with girls and boys of her own age. One of her group, the most vivacious of all, was the girl who prepared the optional related activity on Pythagoras and arranged the special bulletin-board display. She also hoped to become a teacher. The members of this group served as pupil-consultants for other groups when there were more questions than the teacher could care for at a given time.

One boy in group II in the actual distribution of pupil growth was a quiet boy who had been failing geometry since his return from a trip to California, undertaken during the time school had been in session. He had become

discouraged on his return when he found how far the class had advanced in his absence. The unit seemed to revive his interest in geometry, and he was so encouraged by his score in the test that he eventually made up the work he had missed during his absence. There were other interesting facts suggested by figure 3, but space will not permit comments on any other individual records.

Other methods of evaluating the unit.-- It was evident throughout the unit that the class spirit was different than it had been when the work had been conducted in the traditional manner. The teacher-pupil relationship had been pleasant, but a new note was introduced during the unit. The best way to describe the new teacher-pupil relationship would be to compare it with that which exists in well-conducted extra-curricular activities. The pupils accepted the teacher as one of the group. There was no hesitancy about consulting her when they ran into difficulties or felt the need of approval or of suggestions. Moreover, when they consultedthe teacher, they had a clear-cut, definite question or statement to present, showing that they had worked hard before they asked for assistance.

Guidance function.-- The unit on similar triangles served a useful purpose in the field of guidance. The few pupils (Table 16, Appendix A), who had persisted in their choice of the subject in spite of advice to the con-

trary, found that with all the assistance available to them they still were unable to progress very far or very fast. There was no way for them to rationalize their lack of success. Such pupils could see that with all conditions in their favor, their best efforts were not enough to prepare them to meet the demands of the test.

Pupil reactions.-- The class spirit mentioned previously was the composite of the individual reactions to the unit. It could be sensed from the way the pupils carried themselves, their tone of voice, and their purposeful air that they appreciated the responsibility which had been delegated to them. They were natural and they were businesslike. There was an air of self-reliance and self-respect about them. When they had occasion to move about the room they moved with dispatch.

There was more of a demand on the individual pupil in the unit method than in the traditional classroom method. During the laboratory phase when much of the work was done by groups, as explained in chapter III, pupils had to work hard in order to give a good account of themselves in the discussions which went on amongst individual members of groups. No pupil had been appointed or elected as a group leader but gradually certain boys and girls were accorded the position of leadership through mutual recognition of their ability.

Pupils responded well to the chance to do long-range planning and to face difficulties which appeared. In contrast to turning out a set amount of work each class period and dismissing it from their minds at the sound of the bell, pupils divided assignments of their own choosing from the study guide amongst members of their groups so that the work could be carried to completion during study-periods or at home. The work became a part of the pupils' thinking and was discussed in the corridors, home rooms, and in the cafeteria. It was evidently discussed at home, judging from the way in which parents and older brothers and sisters donated books and other material.

When pupils chose their own exercises from the activity guide, they persisted in their efforts to solve them. Discussions within groups were sometimes spirited. When a pupil had proof to defend, it had come as the result of serious endeavor on his part and what the others thought of it was important to him. It is one thing to learn of the general nature of deductive proof and to use it in an academic situation. It is quite a different matter to choose a difficult exercise for oneself, to concentrate on it so that one is oblivious to everything else, and then to present it to one's group, only to have that group challenge the sequence of steps in the proof. Then there develops a real exchange of thought, and logical sequence

takes on a new meaning for that pupil.

To one who has read chapter III, it is apparent that more on the subject of pupil reaction to the unit was interwoven with the account of the teaching of the unit. For that reason only one more comment will be made on the evaluation of the unit.

Pupil opinion of the unit.-- There was no pupil questionnaire given at the close of the unit on similar triangles, but pupils had given a strong indication of their approval in the zeal with which they had worked and in comments which they had made. Many pupils commented favorably on the use of reference books and on the chance to work in groups during the laboratory phase. Others liked the definiteness of the core material and the freedom in choosing exercises from the many listed in the activity guide.

Another aspect of the unit which the pupils said they liked was the chance to talk informally during the class period with the teacher. The teacher had noticed that pupils who had hesitated to ask questions before the class were quick to air their difficulties when she had joined the small group in which they were working.

The pupils' opinion of the success of the unit was most clearly revealed when a number of pupils near the close of the unit asked if the rest of the year's work was

to be studied in the same way. There were so many pupils inquiring that the day before the final tests, the teacher asked how many were in favor of the new method. With the exception of one or two lazy youngsters who evidently realized that it demanded more effort than they cared to expend, the pupils voted for the unit method.

Concluding comments.-- The writer is conscious of the shortcomings of the first experience with the unit method and has pointed out some of the ways in which the unit may be improved. Dr. Billett in Fundamentals of Secondary-School Teaching[1] has written:

> If local teaching procedures must be improved with the fag ends of time and energy, it is often desirable to reorganize the course into unit assignments of a relatively elementary and imperfect sort. Through annual revisions the course may be lifted to higher and higher levels.

The general impression left with the teacher is that of a classroom humming with activity, of boys and girls who were eager to enter and to start work on their own initiative. There is the recollection of a great deal of hard work done thoroughly and happily, and of progress made in the concepts and abilities listed in the delimitation of the unit. Also, there is the feeling of having been accorded, to a high degree, the trust, confidences, and friendship of the pupils. All these things indicate that the unit on

1/ Roy O. Billett, Fundamentals of Secondary School Teaching, Houghton Mifflin Company, Boston, 1940.

similar triangles was worth while and that not only is it desirable to improve this unit, but that other units in the field of geometry should be attempted.

# PART II

## UNIT ORGANIZATION OF THE TOPIC

## LOGARITHMS

# CHAPTER I

# THE PROBLEM AND ITS BACKGROUND

## Statement of the Problem

The problem.-- The problem in Part II of the paper is to apply more completely than was possible in Part I, the basic principles and procedures set forth in Fundamentals of Secondary-School Teaching[1] and the course in the Unit Method in the Secondary School[2] this time to the unit organization and classroom presentation of the topic logarithms to two eleventh-grade classes in college-preparatory mathematics in a general high school.

## The Pupils to be Taught

The classes.-- The second unit was prepared for two groups of college-preparatory pupils who were taking the eleventh-grade course entitled "college mathematics". The course was elective and included second-year algebra and more advanced theorems and exercises in plane geometry than had been studied in the tenth grade. The first group was classified as 11-1, indicating the best group of the class and the second group as 11-3, indicating the slowest group. Division 11-2 was taught by another teacher and did not study the unit. There were twenty-seven pupils in

1/Loc. cit.
2/Loc. cit.

11-1, fourteen boys and thirteen girls. Twenty were in the eleventh grade, six were in the twelfth, and one was taking post-graduate work. The latter and one of the boys classified as a senior were veterans. The following year, of the other seniors who studied the unit, two were freshmen at Radcliffe College, one was at Mount Holyoke, and one was at Chandler School studying to be a medical secretary.

Each group met five times a week for periods of forty-one minutes in length.

The second group, 11-3, was composed of twenty-five pupils, nineteen boys and six girls. Twenty-two of these pupils were in the eleventh grade. Two veterans who had left school to enlist at the end of their eleventh-grade courses had returned and were enrolled as twelfth-grade pupils. The other pupil was a boy who had been refused admittance to a school of technology after having been graduated from the general course in a different high school. He had enrolled in the present school as a post-graduate to make up the subjects specified for college admission.

One boy was on probation from the juvenile court and often sat with an expressionless face gazing off into space. By the end of the unit he was a normal acting boy again due in great part to the group work. There was a

girl in the class who was engaged and was to be married on her sixteenth birthday which came the day after the unit was completed. Another boy had left school to work for two years, and upon his return had found it hard to concentrate on school work for any length of time. This group presented more of a challenge than did the pupils in 11-1, most of whom had similar backgrounds and interests.

Statistics regarding the pupils.-- Information regarding intelligence quotients, previous marks in mathematics, and grade placements in reading vocabulary, reading comprehension, arithmetic reasoning, arithmetic fundamentals, language, and spelling was found on the six-year records of the pupils on file in the office of the school. The tests which had been given included the Stanford Achievement Test, Pintner, Otis Quick Scoring, Henmon-Nelson, and Pintner-Cunningham. There has been no attempt to distinguish them in the tables. (Tables 8 and 9) The chronological ages used in the tables were as of October first, nineteen hundred forty-six. In 11-1, the ages ranged from fifteen years six months to nineteen years four months with a median age of sixteen years one month. The range in intelligence quotients was from ninety-one to one hundred forty-three, a range of fifty-two, and the median was one hundred twenty-six. The low scores in grade placements were as follows when the pupils were in grade 9.1:

Table 8. Information about Pupils, Preparatory to the Unit on Logarithms

| Pupil | Chronological Age Years and Mos. | Intelligence Quotient | Previous Yearly Averages in Mathematics Grades 7 to 10 Gr. 7 | Gr. 8 | Gr. 9 | Gr. 10 |
|---|---|---|---|---|---|---|
| 1 | 15- 7 | 165 | A | A | A | B |
| 2 | 15-11 | 143 | A | A | A | A |
| 3 | 15-11 | 142 | A | A | B | A |
| 4 | 15- 6 | 139 | A | A | A | A |
| 5 | 15- 7 | 137 | B | B | B | A |
| 6 | 17- 1 | 136 | A | B | A | B |
| 7 | 16- 1 | 135 | A | A | A | A |
| 8 | 15- 8 | 134 | A | A | B | B |
| 9 | 15- 7 | 132 | C | B | C | C |
| 10 | 15- 8 | 131 | A | B | A | A |
| 11 | 16- 2 | 131 | - | - | A | A |
| 12 | 15-10 | 130 | A | A | C | A |
| 13 | 15- 9 | 130 | B | C | C | C |
| 14 | 16- 5 | 130 | A | B | C | C |
| 15 | 16- 1 | 129 | C | A | C | C |
| 16 | 16- 5 | 128 | B | A | B | B |
| 17 | 16- 2 | 126 | B | B | C | C |
| 18 | 16- 3 | 124 | B | B | L | B |
| 19 | 16- 1 | 123 | A | B | B | B |
| 20 | 17- 2 | 123 | A | A | A | A |
| 21 | 15- 9 | 123 | B | B | C | B |
| 22 | 19- 4 | 123 | B | B | B | C |
| 23 | 15- 7 | 123 | C | B | A | B |
| 24 | 17- 3 | 123 | C | C | B | C |
| 25 | 20- 4 | 123 | C | A | B | B |
| 26 | 16- 6 | 122 | A | A | B | B |
| 27 | 15-10 | 121 | B | A | C | B |
| 28 | 16- 1 | 120 | A | A | A | A |

Table 8. (concluded)

| Pupil | Chronological Age Years and Mos. | Intelligence Quotient | Previous Yearly Averages in Mathematics Grades 7 to 10 | | | |
|---|---|---|---|---|---|---|
| | | | 7 | 8 | 9 | 10 |
| 29 | 16- 1 | 120 | C | C | L | C |
| 30 | 16-11 | 120 | C | B | D,A | B |
| 31 | 16- 4 | 119 | A | A | A | B |
| 32 | 16-11 | 119 | - | - | C | C |
| 33 | 15- 8 | 117 | A | C | C | C |
| 34 | 15-10 | 117 | B | A | B | L |
| 35 | 16- 1 | 117 | B | C | C | B |
| 36 | 16- 1 | 117 | C | C | C | L |
| 37 | 17- 0 | 117 | B | B | C | C |
| 38 | 15- 2 | 115 | C | B | C | L |
| 39 | 15- 4 | 115 | - | B | C | C |
| 40 | 20- 0 | 115 | C | L | D,L | L |
| 41 | 16- 4 | 114 | C | B | L | C |
| 42 | 17- 6 | 114 | C | C | C | C |
| 43 | 16- 1 | 112 | C | C | C | C |
| 44 | 15-10 | 111 | B | B | B | C |
| 45 | 16- 5 | 111 | C | C | C | L |
| 46 | 16- 7 | 111 | - | - | C | B |
| 47 | 16- 5 | 110 | B | B | C | L |
| 48 | 15-11 | 108 | A | B | A | A |
| 49 | 17-11 | 107 | Not available | | | |
| 50 | 16- 0 | 101 | B | B | C | B |
| 51 | 19- 3 | 91 | L | C | L | L |
| 52 | 16- 2 | 85 | - | - | C | C |

Table 9. Further Information about Pupils

Preparatory to the Unit on Logarithms

| | Grade Placement When in Grade 9. 1 | | | | | |
|---|---|---|---|---|---|---|
| Pupil | Reading Vocabulary | Reading Comprehension | Arithmetic Reasoning | Arithmetic Fundamentals | Language | Spelling |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) |
| 1 | 13+ | 13+ | 13+ | 13+ | 13+ | 13+ |
| 2 | 13+ | 13.0 | 13+ | 13+ | 13+ | 13+ |
| 3 | 13+ | 13+ | 13+ | 13+ | 13+ | 13+ |
| 4 | 10.9 | 11.6 | 13.0 | 10.9 | 12.9 | 12.0 |
| 5 | 11+ | 11+ | 11+ | 10.0 | 7.6 | 9.3 |
| 6 | 11+ | 11.0 | 11+ | 11+ | 11.0 | 11+ |
| 7 | 13+ | 12.6 | 13+ | 13+ | 13+ | 13.0 |
| 8 | 10.6 | 10.3 | 12.9 | 12.9 | 13+ | 9.3 |
| 9 | 9.0 | 10.0 | 9.3 | 9.0 | 11+ | 8.5 |
| 10 | 11.9 | 12.3 | 13+ | 12.0 | 10.9 | 13+ |
| 11 | None available | | | | | |
| 12 | 13+ | 13+ | 13+ | 12.9 | 13+ | 13.0 |
| 13 | 11+ | 11+ | 11+ | 11+ | 11+ | 11+ |
| 14 | 8.3 | 9.3 | 11+ | 11+ | 7.6 | 11+ |
| 15 | 11+ | 11+ | 11+ | 11+ | 11+ | 11.0 |
| 16 | 11+ | 11+ | 11+ | 11+ | 11+ | 11+ |
| 17 | 13.0 | 13+ | 11.6 | 13+ | 13+ | 12.6 |
| 18 | 11.9 | 13.0 | 12.6 | 11.6 | 12.3 | 11.0 |
| 19 | 12.9 | 13+ | 13+ | 13+ | 12.3 | 12.6 |
| 20 | 11+ | 11+ | 11+ | 11+ | 11+ | 11+ |
| 21 | 11+ | 11+ | 11+ | 10.9 | 11+ | 9.0 |
| 22 | None available | | | | | |
| 23 | 11+ | 11+ | 10.3 | 11+ | 9.3 | 10.9 |
| 24 | 11+ | 10.6 | 11+ | 10.3 | 9.5 | 8.1 |
| 25 | 9.5 | 10.5 | 13.0 | 9.9 | 9.9 | 8.8 |
| 26 | 11+ | 11+ | 11+ | 11+ | 11+ | 11+ |
| 27 | 11.3 | 13+ | 12.6 | 13+ | 13.0 | 13+ |
| 28 | 13+ | 13+ | 10.6 | 13+ | 13+ | 12.6 |
| 29 | 9.8 | 8.5 | 9.8 | 11+ | 10.0 | 8.8 |
| 30 | 10.6 | 9.8 | 11+ | 11+ | 10.8 | 8.1 |
| 31 | 11+ | 11+ | 11+ | 11+ | 11+ | 10.0 |
| 32 | 11+ | 11+ | 6.6 | 7.4 | 7.6 | 11 |

Table 9. (concluded)

| Pupil | Grade Placement When in Grade 9.1 | | | | | |
|---|---|---|---|---|---|---|
| | Reading Vocabulary | Reading Comprehension | Arithmetic Reasoning | Arithmetic Fundamentals | Language | Spelling |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) |
| 33 | 11+ | 11+ | 10.6 | 11+ | 7.4 | 8.8 |
| 34 | 9.0 | 9.5 | 9.8 | 10.3 | - | - |
| 35 | 11+ | 11+ | 11+ | 11+ | 9.5 | 11+ |
| 36 | 11+ | 8.5 | 9.3 | 10.6 | 10.0 | 11.0 |
| 37 | 11+ | 9.8 | 9.8 | 11+ | 10.6 | 7.0 |
| 38 | 11+ | 11+ | 11+ | 11.0 | 10.3 | 8.1 |
| 39 | 7.2 | 10.0 | 9.0 | 11+ | 7.0 | 7.8 |
| 40 | Not available | | | | | |
| 41 | 11+ | 11+ | 11+ | 11+ | 11+ | 11+ |
| 42 | 9.5 | 9.4 | 10.3 | 9.1 | 8.0 | 11.0 |
| 43 | 11+ | 11+ | 11.0 | 9.0 | 9.3 | 8.3 |
| 44 | 8.3 | 8.3 | 11+ | 11+ | 9.5 | 9.3 |
| 45 | 10.6 | 10.6 | 9.3 | 9.5 | 9.5 | 7.2 |
| 46 | 11+ | 11.0 | 9.8 | 8.1 | 11+ | 10.0 |
| 47 | 11+ | 11+ | 11+ | 11+ | 11+ | 10.0 |
| 48 | 11+ | 8.5 | 11.0 | 11+ | 10.9 | 11+ |
| 49 | Not available | | | | | |
| 50 | 7.6 | 10.0 | 11.0 | 11+ | 6.6 | 6.2 |
| 51 | 9.6 | 7.9 | 9.7 | 7.6 | 8.2 | - |
| 52 | 8.5 | 6.4 | 6.0 | 6.0 | 7.2 | 7.8 |

reading vocabulary, 9.0: reading comprehension, 7.9; arithmetic reasoning, 9.3; arithmetic fundamentals, 7.6; language, 7.6; spelling, 8.3. The highest placements in all the foregoing were 13+. (Table 9).

Previous marks in arithmetic for grades seven and eight, for algebra in grade nine, and for geometry in grade ten were all available from office records. The average for the year was given in letter grades, interpreted as follows: A excellent, 90 or higher; B good, or 80 to 89; C fair, or 70 to 79; L low, 65 to 69 or the lowest passing mark; and D, failure or below 65. L was supposed to indicate probable failure in case a pupil continued in a sequential subject. There were five pupils in 11-1 whose marks had been all A's. Ten had had marks of all A's and B's. Five had had one C, the rest A's and B's, three had had B's and C's, one had had all C's, and one had had all L's with the exception of one C in the eighth grade. One pupil's record was incomplete, and the remaining pupil had three B's with an L in algebra.

In 11-3 the ages ranged from fifteen years two months to twenty years four months with a median age of sixteen years two months. (Table 8). The range in intelligence quotients was eighty, from eighty-five to one hundred sixty-five. The median was one hundred seventeen. The low scores in grade placements were as follows: reading

vocabulary 7.2; reading comprehension 6.0; arithmetic reasoning 6.0; arithmetic fundamentals 6.0; language 6.4; spelling 6.0. The highest ratings were each 13+.

Previous marks in mathematics from grade seven to grade ten followed no such well defined pattern as had those for 11-1. The best record was that of three A's and one B. The lowest was one C, and three L's. (Table 8). When two marks appear together for one year, it indicates that the pupil repeated the subject, so the last mentioned pupil above had failed ninth grade algebra and had received a mark of L when he repeated it. Three pupils had done no better than L in ninth grade algebra, and six had received marks of L in geometry indicating that these pupils might be expected to have trouble with the eleventh grade work on the basis of insufficient background in mathematics.

Home background of the pupils.-- The information given in Part I concerning the homes and parents of the pupils applied equally to the pupils for whom the second unit was prepared. In fact, eight of the pupils in 11-1 and ten in 11-3 had, the previous year, participated in the unit on similar triangles.

The parents of the pupils in 11-1 included an accountan, assistant superintendent of a yacht company, appraiser, bank clerk, construction business office manager, dentist, director of sales taxes for the state, draftsman, estimator, federal tax collector, foundry superintendent, industrial

engineer, marine engineer, illuminating engineer, insurance clerk, machine designer, manager of a printing plant, manufacturing stationer, office manager, railroad conductor, railway mail clerk, salesman, superintendent of the public schools of the city, telegrapher, training officer for Veterans' Administration, and vice-president of an oil equipment company.

In 11-3, in addition to some occupations given for 11-1, there were the following other occupations: Army officer, bus driver, clerk, civil engineer, electrician, executive foreman of a machine shop, express packer, funeral director, internal revenue agent, linesman, machinist, life insurance underwriter, Navy lieutenant, track official, and track manager.

Extra-curricular and out-of-school activities.-- Of the fifty-two pupils in both groups only eight listed no extra-curricular activities and these eight worked after school hours. The school activities showed a variety of interests such as band, biology club, cafeteria workers, chorus, class dramatics, forestry club, glee club, girls' club, golf team, history club, library staff, Manet staff (school yearbook), mathematics club, motion picture operators' club, National Honor Society, North Star staff (school newspaper), orchestra, photography club, rifle club for boys and for girls, Spanish club, student council, traffic squad, varsity baseball, basketball, and track.

In addition to holding office in many of the above organizations, the pupils in the two groups included the president and secretary of the eleventh-grade class and the president and treasurer of home rooms.

Out-of-school activities and hobbies listed were: attending boxing matches and movies, automobile maintenance and driving the family car, and in two cases, their own cars, bird-life study, bowling, bicycling, camping, card playing, church dramatic societies, dancing, fishing, gardening, hiking, horse-back riding, listening to music, model airplane making, model trains, piano lessons, pool, radio, reading, sailing, skating, skiing, and swimming. One boy sailed his own seventeen-foot sloop, and one girl studied architecture with a senior girl who was admitted to Massachusetts Institute of Technology in June.

Organizations outside of school in which pupils were active included Boy Scouts, Campfire Girls, Catholic Youth Organization, Centro Cultural Panamericano, Christian Endeavor, City of Boston Glee Club, DeMolay, Girl Scouts, Hi-Y, Junior Achievement, Junior Holy Name Society, Junior Legion Band, Missionary Youth Club, Symphony Club, Rainbow Girls, Tri-Hi-Y, Wing Scouts, Youth Fellowship, Young Peoples Fellowship, and Youth Theater.

Again, as in the school organizations, many pupils held office in the above groups. One boy was secretary of the Norfolk-Suffolk Federation of American Unitarian

Youth. He played varsity baseball and was a member of an American Legion summer team. In a competitive examination he won a scholarship sponsored by the Legion for study at a college in Boston. He was elected president of the student council and of the National Honor Society for his senior year.

A boy who belonged to the city wide symphony club, played the piano there, violin in the school orchestra, and French horn in the school and Junior Legion bands. He gave piano lessons during the late afternoon and early evening hours, sang in the school glee club and in two church choirs as soloist, and had an early morning paper route.

One girl played leading roles in the productions of the Youth Theater and served as manager for two other successful plays. It is surprising how many outside activities are carried on by high school pupils without jeopardizing their success in heavy college-preparatory programs. It also helps to explain one of the popular features of the unit methods, the chance to plan their work themselves to suit their own time schedules.

Colleges for which pupils were preparing.-- The colleges for which pupils were preparing are given in tabular form. There were only six of the fifty-two pupils who were not going to college and three more who were not

sure. Some pupils were planning to apply to three or more colleges. One girl was later accepted by the two colleges to which she had applied and was offered a scholarship by one.

Colleges for which Pupils Were Preparing

| Boys' Choices | Frequency | Girls' Choices | Frequency |
|---|---|---|---|
| Massachusetts Institute of Technology | 9 | Radcliffe | 5 |
| Northeastern | 8 | Regis | 3 |
| B.U. College of Liberal Arts | 4 | Boston University Sargent School | 3 |
| Boston College | 3 | B.U. College of Liberal Arts | 3 |
| Georgetown | 2 | Wellesley | 3 |
| Holy Cross | 2 | Emmanuel | 2 |
| U.S. Coast Guard Academy | 2 | Smith | 2 |
| Wentworth Institute | 2 | B.U. College of Business Administration | 1 |
| Amherst | 1 | Bridgewater S.T.C. | 1 |
| Bowdoin | 1 | Chandler School | 1 |
| Dartmouth | 1 | Tufts (Jackson) | 1 |
| Harvard | 1 | Katherine Gibbs | 1 |
| Massachusetts State | 1 | Mass. College of Pharmacy | 1 |
| Middlebury | 1 | Massachusetts State | 1 |
| Princeton | 1 | Mount Allison | 1 |
| Prague | 1 | Univ. of Southern California | 1 |

| Boys' Choices | Frequency |
|---|---|
| Springfield | 1 |
| Tufts | 1 |
| University of Miami | 1 |
| U.S. Maritime Academy | 1 |
| West Point | 1 |
| Williams | 1 |
| William and Mary | 1 |
| Yale | 1 |

The School

Situation of the school.-- The situation of the school was described in the first unit on pages 10 to 16 , so will not be repeated here.

The classroom.-- There had been no changes made in the classroom since the teaching of the first unit. It was after the completion of the second unit that the new large bulletin board had been installed. The description of the classroom will be found on pages 11 to 14.

Books.-- Each member of the class was supplied with a copy of the textbook for the course.[1] The seventh, eighth, and ninth books on the pupil reference list were school property. There was one copy of each of the first of them and three of the last. The second and third on the list

1/Raleigh Schorling, Rolland R. Smith, John R. Clark, Second-Year Algebra, World Book Company, Yonkers-on-Hudson, New York, 1942.

had been purchased by the teacher, and the others were inspection copies given by the publishers. The teacher had not felt justified in asking the school department to purchase extra copies of the reference books until after she had observed pupil reaction to their use. Later she submitted a request and received three copies each of the five books she asked for. These books arrived after the completion of the unit but were used for review purposes and are available for continued use. In her search for good reference books the teacher discovered that unlike the publication of new geometry books, there was a scarcity of new editions of second-year algebra books. Pupils in the class brought in tables of logarithms and college textbooks which belonged to their fathers or brothers, and one college professor who came in to discuss the work of his son, in the tenth grade, became interested in the display of work already done in the eleventh-grade unit and sent in three more good reference books. It was interesting to note in both units that once started, many things were "added unto them". That the pupils really derived benefit from the reference books was later disclosed in the questionnaire when the most prevalent suggestions for improvement of the unit was to obtain more reference books.

Writing the Unit

Determination of objectives.-- The topic logarithms

was part of the course of study for the eleventh-grade mathematics course. Teachers in the department had been left free to introduce it at any time of the year that they thought appropriate. The writer usually taught the topic as the first topic of the new course for psychological and functional reasons. Pupils enjoyed learning something new to them and moreover were then able to use logarithms as tools in succeeding topics. To allow time for the unit to be prepared, the teaching of logarithms was postponed until March. It was fortunate in one way that the unit had not been taught in September as pupil response was so enthusiastic that the rest of the year's work would have been an anticlimax. Units take time to prepare properly, and the teacher would have been hard pressed for the opportunity to prepare others for that year.

Once the topic had been chosen, the objectives in the course of study, which were in a few brief paragraphs in general terms, were written in the form of a series of short declarative sentences so that each concept or practice was definitely stated.

General statement of the unit.-- At the time when the teacher was a member of the seminar in secondary education and was starting to prepare the unit on logarithms she was also enrolled in a course in elementary statistics.

In scanning the pages of the textbook for the course[1] she noticed that each chapter was headed by a quotation. Being of a curious disposition and also averse to settling down to the real work of starting the home assignment for the course, she read some of the quotations, choosing at random. After particularly enjoying the Lewis Carroll quotation heading the tenth chapter[2] and noting privately that there were other phases of mathematics which some people would like to "boil in sawdust" and "salt in glue", she turned to the second chapter of the book to begin the assignment. However, there was no more work done for a few days because, heading the chapter on computation, Professor Mode had placed the quotation on logarithms[3] which was a translation of the words of John Napier in his Descriptio. This was the springboard from which came the beginning of the general statement and the introduction to the unit.

Delimitation of the unit.-- In writing the delimitation of the unit two things were taken into consideration. They were the pupils' interests, needs, and capacities as re-revealed in the study which had been made of them, and the objectives of the topic logarithms after the objectives

1/Roy O. Billett, "Seminar in Secondary Education," School of Education, Boston University, First Semester, 1946.

2/Elmer B. Mode, Instructor, "Elements of Statistics," School of Education, Boston University, First Semester, 1946.

3/Op. cit., p. 10.

had been rewritten in sentence form. This will be found in chapter II of the unit. Next was written the list of probable incidental and indirect learning products and the optional related activities. The next procedure was to prepare the unit of experience sequence.

Unit of experience sequence.-- When the time came for writing the unit of experience sequence, especially in choosing introductory activities and core activities, the teacher was confronted with the choice between the psychological and logical approaches to the topic. In the course which she had taken in the teaching of mathematics, both procedures had been studied and in her own teaching of the subject she had experimented with the methods with different classes.

The teacher was fortunate enough to see the psychological procedure used by the same professor who had taught the previous course when he was later the instructor in a refresher course given during the war.[1] There were in the class students who had not used logarithms, and the psychological approach was the one chosen by the professor to instruct them. They were taught to use logarithms in computation immediately, and refinements such as interpolation and short cuts were postponed until

1/Ralph Beatley, Instructor, "Mathematics for Teachers," Graduate School of Engineering, Harvard University, 1944.

they had acquired confidence and facility in using logarithms for rough computation. That demonstration turned out to be the deciding factor in the teacher's choice of the method used in the unit sequence. However, she assumes all responsibility for any misinterpretation of the intentions or procedures used in the demonstration teaching of the topic and for any adaptations which she felt should be made in presenting the topic to pupils at the eleventh-grade level rather than at the college-graduate level.

It is to be noted that interpolation in this unit is first taught in the finding of antilogarithms rather than in finding logarithms. This again is a matter of choice for the individual teacher, and the teacher of the unit has used now one method and again the other in her past teaching.

A private school teacher was shocked at the time allotment of three or four weeks. One reason for giving the unit so much time was that the class periods were only forty-one minutes long, and the other was that facility in the mechanical operations with logarithms was not the only purpose of the unit. In the study of logarithms important concepts of plane and solid geometry and of algebra were evolving or were being reviewed, and their application in many fields of activity were being made. A strictly mechanical use of logarithms can be taught in

much shorter time, but the enriched course which the teacher wanted for her pupils, to her, justified the extended time.

Preparing the tests.-- The order of the items in the delimitation of the unit was follwed in preparing the tests. The pre-test was to be used again as the final test, although the pupils were not so informed. Part I consisted of ninety true-false statements, some of which required computation and the use of tables. Part II, to be given the next day, contained thirty more complicated exercises of the completion type which also necessitated computation and the use of tables.

The first test during the course of the unit was to be a compilation of exercises submitted by the pupils, and it turned out to be thirty statements of the completion type. These items were submitted while the unit was in actual operation, so were written on the board rather than mimeographed. The other three tests given during the course of the unit were prepared by the teacher on the ditto machine. They had been compiled to be suitable for the stages in the pupil study guide where they had been mentioned. Each contained fifty items of the completion type.

Introduction to the unit.-- In preparing the introduction which was to be made through a bulletin-board display the writer had consulted the mechanical drawing teacher in regard to a printed copy of the Napier quotation. He

offered to make one, and within a few days he had finished a beautiful copy on white cardboard, ten inches by seventeen inches, lettered in India ink.

In her reading the teacher had encountered references to material produced by Scriptura Mathematica[1] so she had written for information and had purchased a folio which contained a photo engraving of John Napier and a folder containing a biographical sketch and a brief introduction to logarithms. The engraving was ten inches by thirteen inches and each page of the folder was ten inches by fourteen inches. These three articles and a table of logarithms[2] completed the bulletin-board display.

Standing open on the top shelf of the bookcase there was to be a copy of Four-Place Tables[3] clipped open to pages sixteen and seventeen. Also standing on the top shelf, fastened open by elastic bands to pages which showed good logarithmic set-ups and solutions of interesting problems, would be two of the newest reference books, and in the lower shelves the rest of the reference books would be arranged. On the binding of each reference book there was placed a small gummed label which bore the number which

1/David Eugene Smith, Portraits of Eminent Mathematicians with Biographical Sketches, V, Portfolio I, 1946 Edition, Scriptura Mathematica, New York, 1936.

2/Elmer B. Mode, Prentice Hall Mathematical Tables to Accompany the Elements of Statistics, Prentice-Hall Incorporated, New York, 1941, p. 2-3.

3/E. V. Huntington, Four-Place Tables, Unabridged Edition, Houghton Mifflin Company, Boston, 1910, p. 16-17.

preceded the name of the book in the pupil reference list.

Assistance of the commercial department.-- The unit-of-experience sequence, study guides and pre-tests were prepared by students in the commercial department under the supervision of the teacher of office secretarial practice and with the approval of the principal of the school and the heads of the mathematics and commercial departments. There were twelve mimeographed, stapled pages in each copy of the unit-of-experience sequence in which were incorporated the study guides; five pages in each copy of Part I of the pre-test; and three in each copy of Part II. Single spacing was used for the unit-of-experience sequence to save stencils, but the tests were double spaced. The stencils and paper were provided by the commercial department. Four pupils prepared the experience sequence and these four, with the help of three others, did the tests. The most difficult part was the typing of exponents, sub-scripts, and bases in the logarithmic form. Fractional exponents gave particular trouble. The pupils used the standard typewriters, raising and lowering the carriage for exponents and bases. The typing was started in November and was interrupted a few times by urgent work, such as Christmas programs and examinations for heads of departments, all this in addition to the regular work of the pupils.

The teacher of the unit helped the pupil-typists to interpret mathematical symbols and in turn learned much

about preparing stencils. The commercial and mathematics departments developed an increased respect for one another. The pupil-secretaries were impressed by what the mathematics pupils were expected to learn; the mathematics teacher and later the pupils were appreciative of the ability of the commercial pupils to prepare such attractive and accurate copies of difficult material. Without this mimeographed material, the unit would have been much less pleasantly and efficiently carried out.

Summary of preparations.-- Preparations for teaching the unit on logarithms were completed on Friday, March seventh. When the pupils had left the building, the bulletin-board material was arranged, and the bookcase exhibits and reference books were in place. There were two piles of mimeographed material on the circular table at the front of the room, the unit-of-experience sequence and Part I of the pretest. The first of these contained core material, references to optional related activities, predetermined instruction points, and study guides. The small box which contained the library cards on which were printed directions for optional related activities was also on the table. Special books, some of them school library books, reserved for the optional related activities were in the glass-fronted bookcase at the front of the room.

In the closet at the rear of the room was a carton which served as a filing cabinet. In it, placed sidewise

and separated by oak card indices, were the other tests, mimeographed copies of Part II of the pre-test and ditto machine copies of tests two, three, and four. Everything was in readiness for the actual introduction of the unit.

Table 10. Scores on Logarithm Test

| Pupil | Pre-Test Part I | Pre-Test Part II | Pre-Test Total | Final Test Part I | Final Test Part II | Final Test Total | Gain |
|---|---|---|---|---|---|---|---|
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| 1 | 19 | 0 | 19 | 77 | 24 | 101 | 82 |
| 2 | 31 | 0 | 31 | 79 | 17 | 96 | 65 |
| 3 | 12 | 0 | 12 | 78 | 22 | 100 | 88 |
| 4 | 12 | 1 | 13 | 76 | 20 | 96 | 83 |
| 5 | 23 | 5 | 28 | 75 | 19 | 94 | 66 |
| 6 | 14 | 0 | 14 | 68 | 19 | 87 | 73 |
| 7 | 47 | 0 | 47 | 84 | 26 | 110 | 63 |
| 8 | 18 | 0 | 18 | 74 | 20 | 94 | 76 |
| 9 | 6 | 0 | 6 | 69 | 14 | 83 | 76 |
| 10 | 52 | 0 | 52 | 71 | 21 | 92 | 40 |
| 11 | 48 | 0 | 48 | 85 | 26 | 111 | 63 |
| 12 | 7 | 0 | 7 | 69 | 19 | 88 | 81 |
| 13 | 6 | 0 | 6 | 59 | 14 | 73 | 67 |
| 14 | 47 | 0 | 47 | 61 | 16 | 77 | 30 |
| 15 | 0 | 0 | 0 | 41 | 10 | 51 | 51 |
| 16 | 16 | 0 | 16 | 73 | 17 | 90 | 74 |
| 17 | 7 | 0 | 7 | 72 | 21 | 93 | 86 |
| 18 | 38 | 0 | 38 | 66 | 17 | 83 | 45 |
| 19 | 24 | 0 | 24 | 69 | 14 | 83 | 59 |
| 20 | 6 | 2 | 8 | 82 | 21 | 103 | 95 |
| 21 | 20 | 0 | 20 | 71 | 17 | 88 | 68 |
| 22 | 58 | 20 | 78 | 81 | 23 | 104 | 26 |
| 23 | 44 | 0 | 44 | 67 | 18 | 85 | 41 |
| 24 | 6 | 0 | 6 | 70 | 17 | 87 | 81 |
| 25 | 21 | 0 | 21 | 70 | 19 | 89 | 68 |
| 26 | 9 | 0 | 9 | 74 | 24 | 98 | 89 |
| 27 | 19 | 1 | 20 | 70 | 15 | 85 | 65 |
| 28 | 36 | 0 | 36 | 79 | 14 | 93 | 57 |

Table 10 (concluded)

| Pupil | Pre-Test Part I | Pre-Test Part II | Pre-Test Total | Final Test Part I | Final Test Part II | Final Test Total | Gain |
|---|---|---|---|---|---|---|---|
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| 29 | 19 | 0 | 19 | 70 | 11 | 81 | 62 |
| 30 | 48 | 0 | 48 | 72 | 22 | 94 | 46 |
| 31 | 16 | 0 | 16 | 67 | 20 | 87 | 71 |
| 32 | 38 | 0 | 38 | 61 | 13 | 74 | 36 |
| 33 | 45 | 0 | 38 | 62 | 15 | 77 | 32 |
| 34 | 12 | 0 | 12 | 65 | 19 | 84 | 72 |
| 35 | 10 | 0 | 10 | 77 | 17 | 94 | 84 |
| 36 | 26 | 0 | 26 | 58 | 14 | 72 | 46 |
| 37 | 0 | 0 | 0 | 73 | 16 | 89 | 89 |
| 38 | 7 | 0 | 7 | 57 | 15 | 72 | 65 |
| 39 | 14 | 0 | 14 | 69 | 17 | 86 | 72 |
| 40 | 51 | 2 | 53 | 70 | 21 | 91 | 38 |
| 41 | 13 | 0 | 13 | 67 | 14 | 81 | 68 |
| 42 | 18 | 0 | 18 | 69 | 19 | 88 | 70 |
| 43 | 43 | 1 | 44 | 67 | 17 | 84 | 40 |
| 44 | 23 | 1 | 24 | 71 | 17 | 88 | 64 |
| 45 | 9 | 0 | 9 | 62 | 16 | 78 | 69 |
| 46 | 8 | 0 | 8 | 60 | 12 | 72 | 64 |
| 47 | 51 | 0 | 51 | 62 | 14 | 76 | 25 |
| 48 | 28 | 7 | 35 | 79 | 21 | 100 | 65 |
| 49 | 18 | 0 | 18 | 57 | 14 | 71 | 53 |
| 50 | 30 | 0 | 30 | 52 | 16 | 68 | 38 |
| 51 | 51 | 0 | 51 | 63 | 17 | 80 | 29 |
| 52 | 15 | 1 | 16 | 61 | 9 | 70 | 54 |

*Seeing there is nothing (right well beloved students in the Mathematickes) that is so troublesome to Mathematicall practice, nor that doth more molest and hinder Calculators than the Multiplications, Divisions, square and cubical Extractions of great numbers, which, besides the tedious expense of time, are for the most part subject to many slippery errors ... and having thought upon many things to this purpose, I found at length some excellent briefe rules.*

*John Napier,*
*Mirifici Logarithmorum Canonis Descriptio.*
*1614.*

## CHAPTER II

## UNIT ORGANIZATION OF THE TOPIC

## LOGARITHMS

### General Statement of the Unit

General statement of the unit.-- The general statement of the unit is taken from the writing of John Napier,[1] the inventor of logarithms. Early in the seventeenth century he wrote:

> Seeing there is nothing (right well beloved students in the Mathematickes) that is so troublesome to Mathematicall practice, nor that doth more molest and hinder Calculators than the Multiplications, Divisions, square and cubical Extractions of great numbers, which, besides the tedious expense of time, are for the most part subject to many slippery errors.. and having thought upon many things to this purpose, I found at length some excellent briefe rules.

Eleventh-grade pupils in college-preparatory classes in mathematics can use logarithms to advantage in their work in numerical trigonometry, in geometric formulas, in biology, chemistry, and physics.

Pupils need some understanding of the theory underlying computation by logarithms, aptness in using the language of logarithms, and facility in the use of tables of common logarithms. It is important that they know

1/John Napier, Mirifici Logarithmorum Canonis Descriptio, 1614. Quoted from Elmer B. Mode, The Elements of Statistics, Prentice-Hall, Incorporated, New York, 1945, p. 10.

how to set up a problem logically and that they know when and when not to use logarithms.

Special fields of work, beyond the high school level, call for a knowledge of logarithms. One of these fields is engineering.

Nelson M. Cooke,[1] in Mathematics for Electricians and Radiomen, writes that although the slide rule meets the ordinary demands for accuracy in problems relating to electricity and radio, the electrical engineer and especially the communication engineer, should have a thorough understanding of logarithmic processes. Many situations require a higher degree of accuracy than can be secured by muse of the slide rule.

Delimitation of the Unit

1. Logarithms were invented as "some excellent briefe rules" or as labor-saving devices by John Napier, a baron in Scotland, about 1614.
2. He first worked out a system to simplify multiplications involving sines in trigonometry.
3. Later the work was extended to operations with numbers in general.
4. The theory of exponents was not known at that time, so Napier did not use a base for his system.

1/Nelson M. Cooke, Mathematics for Electricians and Radiomen, First Edition, McGraw-Hill Book Company, Incorporated, New York, 1942, p. 279.

5. Napier's friend, Henry Briggs, an English professor of mathematics and science, worked out tables of logarithms to the base ten. They are called tables of common logarithms.
6. Logarithms are used to multiply numbers, divide them, raise numbers to powers, and find roots of numbers.
7. Addition and subtraction cannot be done by logarithms.
8. At the beginning of the unit logarithms are considered as exponents. In the equation $4^2 = 16$, 4 is the base, 2 is the logarithm, and 16 is the number. The numerical form may be replaced by words, that is $(\text{base})^{\log} = \text{number}$.
9. If the base is 2 and the logarithm is 5, then the number is 32 and the equation would appear as $2^5=32$.
10. A table of powers of 2 is useful for multiplying or dividing numbers such as 4096, 32768, or other numbers which are expressed in such a table.

$$\frac{32768}{4096} = \frac{2^{15}}{2^{12}} = 2^3 = 8$$

11. The use of such a table is limited because the powers of 2 have not been worked out to include many numbers.
12. The table of positive numbers expressed as powers of 10 has been referred to before and is available in algebra text books or in separate books. See

the table of Common Logarithms,[1] 472-473 in the text: Schorling, Clark, Smith, Second-Year Algebra.

13. The numbers are found in the margins of the tables, in the left-hand and upper margin. The lower margin merely repeats the numbers in the upper for convenience when working with the lower part of the page.

14. The four digit numbers occupying the rest of the table are parts of logarithms.

15. It is important to know why we say: "Parts of logarithms". The reason will be apparent when the table has been used a few times.

16. The base is 10 in this table of common logarithms. (See No. 12)

17. It is possible by using the table to write any positive number, such as 257, to the base 10. This means that 10, raised to a certain power, equals 257.

18. The first two digits, 2 and 5, are found in the left margin on page 472. It is helpful if a piece of paper is placed below the 25 so that it fits crosswise below the numbers in the 25 row. All decimal points in the margin numbers may be ignored

1/Raleigh Schorling, Rolland Smith, and John R. Clark, Second Year Algebra, Schorling-Clark-Smith Mathematics Series, World Book Company, Yonkers, New York, 1942, pp. 472-473.

at this part of the work. The third digit 7, of 257, is found in the upper margin. This column is followed down until the paper beneath the 25 row is reached. This gives 4099.

19. Common sense tells us that $10^{4099}$ power cannot equal 257. Since $10^2 = 100$, it can be inferred that $10^{2.4099}$ is more likely to equal 257.

20. The logarithms given in the table are the decimal parts of the logarithms. They are called the mantissas.

21. The part of the logarithm that tells the next lowest even power of 10 is called the characteristic. (See Item 15)

22. Thus, a logarithm like 2.4099 is made up of an integral part, 2, and a fractional part, .4099. The characteristic is 2 and the mantissa is .4099. The characteristic should always be written before the mantissa is looked up.

23. To multiply 394 by 49.7 by logarithms, the exercise may be rewritten: $10^{2.5955} \times 10^{1.6964} = 10^{2.5955+1.6964} = 10^{4.2919}$. This is because of the algebraic principle that $a^m \times a^n = a^{m+n}$. Since $10^4 = 10{,}000$, $10^{4.2919}$ is a number in the 10,000's. Since 394 is approximately 400 and 49.7 is approximately 50, 394 x 49.7 will be approximately 20,000, which is a number in the 10,000's.

24. The number corresponding to a certain logarithm may be found by using the table and searching in the mantissas for the nearest mantissa to the one in the exercise, in this case to 2919. The table contains 2900 and 2923. Since 2923 is the nearer of the two to 2919, a paper may be slipped below the 2923 row, and the numbers in the margin to the left and in the upper margin copied, as shown. To the left is 19 and above is 6, so the digits in the number corresponding to $10^{4.2919}$ are 196. The estimated answer was 20,000, or a number in the 10,000's. To make 196 conform to this, it is necessary to change it to 19,600.
25. When compared with the answer 19581.8, it is seen that the logarithmic answer is correct to three significant digits.
26. By means of interpolation, to be learned later, the result will be found to be 19,580, which is correct to four significant digits.
27. A four-place table will give results correct to four significant digits.
28. A five-place table is used in the twelfth-grade course, and gives results correct to five significant digits.
29. Three-digit accuracy is sufficient for the first four or five lessons for eleventh-grade students,

or until they have acquired confidence in using the table and in understanding the vocabulary of logarithms.

30. In using any base except 10 it is necessary to write the base. In the notation $2^3 = 8$, 2 is the base, 3 is the logarithm, and 8 is the number. See item 9.

31. The above is called the exponential form.

32. The same idea may be written: The logarithm of 8, to the base of 2, is 3. The logarithm, or the exponent, is plainly seen to be 3.

33. A shorter way to write the expression is $\log_2 8 = 3$. The log = 3. It is the log of 8, that is, log 8 = 3. Since it is necessary to know the base, the base 2 is writeen in as shown, $\log_2 8 = 3$.

34. $\text{Log}_2 8 = 3$ is called the logarithmic form.

35. It helps pupils to understand logarithms when both forms are stressed.

36. Since common logarithms are logarithms to the base 10, it is not necessary to write the base when using the logarithmic form after pupils clearly understand the procedure.

37. $18 = 10^{1.2553}$ may be written $\log_{10} 18 = 1.2553$, or merely log 18 = 1.2553.

38. $\frac{3960}{29}$ may be written $\frac{10^{3.5977}}{10^{1.4624}} = 10^{3.5977-1.4624} =$

$10^{2.1353}$ = approximately 137. The algebraic principle involved is $\frac{a^m}{a^n} = a^{m-n}$.

39. An easier way is to use the logarithmic form:

| | |
|---|---|
| Log 3960 | = 3.5977 |
| Log 29 | = 1.4624 Subtract |
| Log of the quotient | = 2.1353 |
| The quotient | = 137 approximately |

40. If $\log_{10} N = 3.7474$, it may be thought of as $N = 10^{3.7474}$. The characteristic 3 indicates that N, the number, is at least $10^3$, or 1000. The mantissa 7474 indicates that N is greater than 1000, in fact, nearer to $10^4$, or 10,000. Thus N is greater than 1,000 and less than 10,000. The mantissa 7474, in the table, leads to the number 559, so N = 5,590.

41. There is a rule of thumb for finding the characteristic of the log of a number and another for finding the number of digits in the antilogarithm.

The characteristic of the log of a number is one less than the number of digits in the integral part of the number. Thus, the characteristic of log 723.5 = 2, of log 25.7 is 1 and of log 2,426,000 is 6. This may be checked by the original method of finding characteristics.

The number corresponding to a given logarithm is called the antilogarithm.

In No. 40, it was found that if log N = 3.7474, N = 5590.

42. To multiply 295 x 295 by logarithms, the procedure is as follows:

Let N = 295 x 295

Log N = log 295 + log 295

log 295 = 2.4698

log 295 = 2.4698 (Add)

log N = 4.9396

N = 8,7000 (Without interpolating)

By observation it is obvious that instead of adding the logarithms, it is possible to multiply the log of 295 by 2.

Thus: $295 \times 295 = 295^2$

Log N = 2 log 295

= 2 (2.4698)

= 4.9396

N = 87,000

The foregoing is especially useful in cases of powers greater than the second, as will be apparent in compound interest problems and in geometric series.

43. By analogy, if $N = 35.6^{\frac{1}{2}}$, then log N = $\frac{1}{2}$ log 35.6,

and if $N = 234^{1/3}$, $\log N = 1/3 \log 234$. Thus it is possible to find square roots and cube roots by logarithms.

44. It is seen by experience with logarithms that[1] "the logarithm...of a given number...is an exponent indicating the power to which another number, called the base, must be raised in order to equal the given number."

45. Numbers between 0 and 1 have negative characteristics. This is apparent when pupils and teacher construct a brief table of powers of 10. The table may be started with $10^1 = 10$ and progress to $10^3 = 1000$; then in the other direction to $10^{-3}$. Finally, $10^{\frac{1}{2}}$, $10^{\frac{1}{4}}$, $10^{3/4}$, and ten to the other fractional powers may be computed as shown in Table 1: Table of Powers of 10

| | | | |
|---|---|---|---|
| $10^{-3}$ | .001 | $10^{1\frac{1}{4}}$ | 17.78 |
| $10^{-2}$ | .01 | $10^{1\frac{1}{2}}$ | 31.62 |
| $10^{-1}$ | .1 | $10^{13/4}$ | 56.25 |
| $10^{0}$ | 1.0 | $10^{2}$ | 100.00 |
| $10^{\frac{1}{4}}$ | 1.778 | $10^{2\frac{1}{4}}$ | 177.8 |
| $10^{\frac{1}{2}}$ | 3.162 | $10^{2\frac{1}{2}}$ | 316.2 |
| $10^{3/4}$ | 5.625 | $10^{23/4}$ | 562.5 |
| $10^{1}$ | 10.000 | $10^{3}$ | 1000. |

1/ Op. cit., p. 375

The value $10^{.50} = 10^{\frac{1}{2}} = \sqrt{10} = 3.162$ is the first fractional value to be found. Then $10^{0.25}$ may be computed as follows: $10^{\frac{1}{4}} = (10^{\frac{1}{2}})^{\frac{1}{2}} = \sqrt{3.162}$. The value of $10^{0.75}$ may be found by expressing $10^{3/4}$ as $10^{\frac{1}{2}} \times 10^{\frac{1}{4}} = 3.16 \times 1.78$. The other items in the table may be figured in a similar manner.

The definition of $a^{-m} = \frac{1}{a^m}$ is used in the case of the negative exponents.

46. When a logarithm is negative, as in $10^{-3.00}$, the characteristic only is negative. All mantissas are positive. Log $10^{-1.5391}$ means that the characteristic 1 is negative, but the mantissa .5391 is positive. Thus the logarithm given above is actually -1 + .5391.

47. To make it easier to compute with such logarithms they are written as follows: -1.5391 = 9.5391-10; -2.5391 = 8.5391-10; and -3.5391 = 7.5391-10.

48. By comparison with the pupils' table of powers of 10, a rule of writing characteristics for fractions may be formulated: log .14 = 9.----10, log .014 = 8 ....-10; log .0014 = 7.----10. A rule is that a characteristic of -1, or 9....-10 belongs to a decimal where the first significant digit is in the first decimal place. If the first significant digit is in the second decimal place, the characteristic is -2 or 8....-10.

49. Conversely, by reference to the table, it is seen that if log N = 9.8109-10, the value of N is .647. If log N = 8.8109-10, N = .0647. By observation, a characteristic of -1 or -2, indicates that the antilogarithm is a decimal with the first significant digit in the first or second decimal place, respectively.

50. Computation with logarithms should not be postponed until the understanding is complete, but should begin with item 10 in the delimitation. Understanding and use should progress together, new concepts being supplied when their lack impedes progress.

51. The logarithmic curve, $y = \log_{10} x$, adds to the understanding of logarithms. It is apparent, from the graph, that a negative number does not have a real logarithm. The logarithmic curve approaches the y-axis but does not reach it, thus ruling out any attempt, at this stage, to express the log of zero. The logarithm of a number between 0 and 1 is negative, the logarithm of 1 is 0, and the logarithm of 10 is 1. The logarithm of a number greater than 1 is positive.

52. The foregoing use of the graph is an exemplification of previously acquired concepts.

53. The logarithmic curve has a new use as an introduction to interpolation.

> As a number increases, its logarithm increases. Equal changes in the number do not produce equal changes in the logarithm; that is, the increase in the logarithm of a number is not proportional to the increase in the number. However, for small changes in the number, the increases are approximately proportional and are so considered in the following section on interpolation.[1]

54. Logarithms which are between the logarithms given in the table may be found by a process called interpolation.

Log 2640 is given as 3.4216 and log 2650 is given as 3.4232, so one may expect log 2645 to be between log 2640 and log 2650. Assuming that the increase is approximately proportional, as explained above, it is natural to expect the log to be about halfway between 3.4216 and 3.4232 since 2645 is halfway between 2640 and 2650.

The distance between 3.4216 and 3.4232 is 0.0016, half of the distance is 0.0008, so 3.4216 is increased by 0.008. Thus, log 2645 is 3.4224.

55. The computation is usually done using mantissas only as follows:

Mantissa 2640 = .4216

2645 = .4224

2650 = .4232

1/ Op. cit., p. 383

The following computation is written to one side or on scrap paper and the result added to $.4216\ \frac{.5}{10} \times 16 = 8$. This is actually .0008. When added to .4216, the resulting .4224 is the mantissa which was missing from the table.

Therefore, log 2645 = 3.4224.

56. Since a four place table is being used, corrections are rounded off to the nearest digit. Find log 75.78. The characteristic is 1. The mantissa is between the mantissas of 7570 and 7580. The work follows:

Mantissa 7570 = 8791

7578 = 8796 $\quad \frac{8}{10} \times 6 = 4\ \frac{4}{5}$ or 5

7580 = 8797 $\quad$ 8791 5 = 8796

Log 75.78 = 1.8796. Decimal points may be omitted in the computation since they affect the characteristic and not the mantissa.

Many pupils prefer to use decimal fractions instead of common fractions for the computation. The work would then be: .8 x 6 = 4.8 or 5.

57. Antilogarithms may be found to four significant digits. If log N = 1.9568, it is seen that the mantissa 9568 is not in the table. However, 9566 and 9571 are given, so 9568, which is between them, may be found by interpolation. The table shows

that 9566 is the mantissa of the number 9050 and 9571 is the mantissa of the number 9060. The work may be arranged as follows:

9566 leads to 9050

9568 leads to 9054 (This space is at first left vacant as before)

9571 leads to 9060

$\frac{2}{5} \times 10 = 4$. Therefore, if log N = 1.9568, N = 90.54.

58. A plan or set-up should be written before the table is used. Characteristics should be included, as shown below.

Find the radius of the largest gas tank that could be placed on a triangular plot 77.53 feet, 98.47 feet, and 84.03 feet, respectively. By geometry $r = \frac{K}{s}$, where K is the area of the triangle, s is one-half the perimeter and r is the radius of the inscribed circle.

Since $K = \sqrt{s(s-a)(s-b)(s-c)}$, when a, b, and c are the sides of the triangle, the formula becomes $r = \sqrt{\frac{s(s-a)(s-b)(s-c)}{s}}$, or $r = \sqrt{\frac{(130.0)(52.49)(31.55)(45.99)}{130.0}}$

Log 4 = $\frac{1}{2}$ (log 130.0 + log 52.49 + log 3.55 + log 45.99)-log 130.0.

log 130.0 = 2.

log 52.49 = 1.

log 31.55 = 1.

log 45.99 = 1._____(Add)
$\frac{1}{2}$ (  ) =

log 130.0 = 2._____(Subtract)

log r =

r =

The completed work is shown below:

log 130.0 = 2.1139 mantissa 5240 = 7193
5249 = 7201
log 52.49 = 1.7201 5250 = 7202

log 31.55 = 1.4990 9 x 9 = 8.1 or 8

log 45.99 = 1.6627(Add) mantissa 3150 = 4983
$\frac{1}{2}$ (6.9957) 3155 = 4990
= 3.4979 3160 = 4997
= 2.1139(subtract) .5 x 14 = 7.0

log r = 1.3840 mantissa 4590 = 6618
r = 24.21 feet 4599 = 6627
4600 = 6628

.9 x 10 = 9.0

3838 leads to 2420
3840 leads to 2421
3856 leads to 2430

$\frac{2}{18}$ x 10 = 1 1/9 or 1

59. If the logarithm of the subtrahend is larger than the logarithm of the minuend computation is facilitated by adding and subtracting ten, as shown.

11.1426-10
2.2387 (subtract)

The minuend has been changed from 1.1426 to 11.1426-10, that is, the characteristic 1 has become 11-10.

60. When 1/3 log r would lead to the situation

1/3 (9.3979-10) or 3.1326-3 1/3, the expression may be changed to the more convenient 1/3 (29.3979-30) or 9.7993-10. To the 9.....-10 there has been added 20.....-20 so that the -10 there has become a multiple of 3.

61. When an exercise involving negative numbers is to be solved by logarithms, it is essential to remember that negative numbers do not have logarithms. To solve this difficulty, it is possible to first determine the sign of the result by using the law of signs which applies, and then proceed as though the numbers were all positive.

$$\sqrt[3]{\frac{2.87}{-.535 \times 8.37}}$$

The sign of the denominator is negative, so the sign of the fraction is negative. The cube root is negative. Once the sign has been determines, the exercise can be solved as:

$$\sqrt[3]{\frac{2.87}{.535 \times 8.37}}$$

62. Logarithms are time saving in solving problems which involve formulas when the processes indicated are multiplication, division, raising to powers and finding roots.

63. Although addition and subtraction cannot be done by logarithms, there are situations where arithmetic and logarithms are used in conjunction. To

find the hypotenuse, c, of a right triangle, the squares of the lengths of the arms may be found by logarithms, the sum of the squares found by addition, and the square root found by logarithms.

Find c if the arms of the right triangle are 3.75 units and 5.88 units respectively.

$c = \sqrt{3.75^2 + 5.88^2}$

Let $3.75^2 = x$ and $5.88^2 = y$

$$\begin{aligned} \text{Log } x &= 2 \log 3.75 & \text{Log } y &= 2 \log 5.88 \\ &= 2(0.5740) & &= 2(0.7694) \\ &= 1.1480 & &= 1.5388 \\ x &= 14.06 & y &= 34.58 \end{aligned}$$

$$\begin{aligned} c &= \sqrt{14.06 + 34.58} \\ &= \sqrt{48.64} \end{aligned}$$

$$\begin{aligned} \text{Log } c &= \tfrac{1}{2}\log 48.64 \\ &= \tfrac{1}{2}(1.6870) \\ &= 0.8435 \\ c &= 6.972 \\ c &= 6.97 \end{aligned}$$

64. [1] Problems in compound interest illustrate the efficiency of the use of logarithms...If a sum of p dollars is invested at an interest rate 4, compounded k times a year for n years, the amount A is given by the formula,

$$A = p(1 + \frac{r}{k})^{kn}$$

...For greater accuracy tables containing more significant figures must be used. Tables are available to as many as twenty places. In practice, the type of problem determines the kind of table used.

1/ Op. cit., p. 390

This formula can also be used to find the principal p which must be invested at an interest rate r compounded k times a year for n years to amount to any given figure A. In this case p is called the present value of A.

1/ "Find the present value of $5000 to be used in 18 years to send a boy to college, interest being compounded at 3 % semiannually."

The set-up for the preceding exercise would be the following:

$$5000 = p\left(1 + \frac{.03}{2}\right)^{36}$$

$$5000 = p\,(1.015)^{36}$$

$$p = \frac{5000}{1.015^{36}}$$

$$\log p = \log 5000 = \log 1.015^{36}$$

$$\log 5000 = 3.6990$$

$$36 \log 1.015 = 36(0.0065) = \underline{0.2340} \quad \text{Subtract}$$

$$\log p = 3.4650$$

$$p = \$2917$$

65. Exponential equations which cannot be solved by inspection may be solved by logarithms. The answers will be only approximate as the solutions are rational in special cases only. In the compound interest formula, solving for n leads to an exponential equation.

66. Logarithms will be useful in many of the new topics to be taken up in the eleventh and twelfth grade work in mathematics.

1/ Ibid. p. 391.

67. A problem in trigonometry may call for the solution of the following: $\sqrt{\dfrac{0.3147}{6.154 \cos 32^\circ 20'}}$

There is a special table giving the logarithms of trigonometric functions. At this stage of the work the above exercise may be solved by solving

$$\sqrt{\frac{0.3147}{6.154 \times .8450}}$$

The method of interpolation learned in logarithms will be used in trigonometry and in courses in statistics.

68. In using the binomial theorem the occasion may arise where it is necessary to find $(0.3x)^{16}$.

69. The n th term of a geometric series, another new topic, introduces the formula $l = a\, r^{n-1}$, which may require the logarithmic solution of $600(1.02)^{29}$.

70. Logarithms are handy tools in many exercises[1] in geometry. The formula $V = \frac{4}{3} \pi r^3$ represents the volume of a sphere of radius r. Find the radius of the sphere whose volume is 81.28 cubic inches. Take $\pi = 3.142$. The equation will become $r = \sqrt[3]{\dfrac{3(81.28)}{4(3.142)}}$ which can be solved by logarithms.

71. Subjects other than mathematics[2] require the use of logarithms. The text book gives the following

1/ David P. Smith, Jr., Leslie T. Fagan, Mathematics Review Exercises, Ginn and Company, Boston, 1940, p. 41.

2/ Op. cit., p. 389

exercise: PV = RT is a formula which occurs in chemistry, with values such as P = 760.8, V = 22.40, and T = 288.7 where P represents pressure in millimeters, V represents volume, and T represents absolute temperature of C + 273.

72. Chemistry teachers have mentioned the use of logarithms in connection with Ph values. A value of 7 indicates "absolute neutral", a value above 7 indicates alkaline, and below 7, acid.

73. Aviation is another field in which logarithms are useful. A design problem[1] may require the velocity of a plane when the gross weight is changed and when the wing area is changed. Two problems are involved, with an appropriate formula for each. The two solutions can be combined as follows:

$V_2 = V_1\sqrt{\frac{W_2 x\ S}{W_1 x\ S_2}}$, where $V_1$ = given landing speed, $V_2$ = new landing speed, $S_1$ = given wing area, $S_2$ = new area, $W_1$ = gross weight, and $W_2$ = new gross weight.

74. The study of electricity contains formulas which lend themselves to logarithmic solution.[2] H P to drive fans = $\frac{K \text{ x cu.ft.min. x water gage press.in in.}}{33000 \text{ x Eff. of fan}}$

Water gage inches = 1.728 oz. per sq. inc., K = 5.2, Eff. = .5 for ordinary fans to .65 for Sirocco type.

1/George Osteyee, Mathematics in Aviation, Air-Age Education Series, The Macmillan Company, New York, 1942, p.112.

2/Westinghouse Electric and Manufacturing Company, Westinghouse Electric Notes and Data Book, Westinghouse, East Pittsburgh, Pennsylvania, 1936, p. 22.

75. The physics courses yield a wealth of formulas which can be solved by logarithms. The pendulum formula $t = \sqrt{\frac{\pi l}{g}}$, when $\pi = 3.142$, $l = 50.62$, and $g = 980.0$ is one such instance.

76. The slide rule is based on logarithms.[1] "This instrument may be described briefly as a mechanical logarithm table."

Unit Assignment

(Tentative Time - Three or Four Weeks)

The unit assignment differed from the pupils' study guide only in that it included the introduction. The code system used for the pupil study guide made it possible for it to be used by teacher and pupils. In the interests of economy of space the pupil study guide will be given instead of the unit assignment, thus avoiding the repetition of twelve or so identical pages.

Pupils' Study Guide

Core activities are numbered and are to be done by all pupils. Lettered items, preceded by an asterisk, such as *a, are optional related activities and may be found catalogued in the card case. They may be done with the approval of the teacher. "Predetermined instruction point"

1/Elmer B. Mode, The Elements of Statistics, Prentice-Hall, Incorporated, New York, 1941, p. 23.

is a reference for the teacher and means that this work will be developed by the teacher and pupils together. However, you need not wait for these lessons but may work ahead at your own speed if you wish.

The notations T and R-1 refer to the Pupils' Bibliography. T: 376:36-48 refers to exercises 36 to 48 inclusive on page 376 of your text book. R-5: 375:1-9 refers to exercises 1 to 9 inclusive on page 375 of the fifth reference book. Consult the classroom library rules in regard to the use of these reference books.

Books reserved for the optional related activities are listed separately and may be obtained by consulting the teacher.

1. How did logarithms come into existence? For what purposes are they used? What two men in particular are remembered in connection with logarithms? When were logarithms invented? Compare the date with the date of the arrival of the Pilgrims at Plymouth. T: 393; R-5: 389; R-9: 202.

*1a

I Predetermined instruction point: 2-5.

2. Think of a logarithm as an exponent. You know that $4^2 = 16$. Think of 2 as the logarithm. You know that 4 is the base. Call 16 the number. Fill in the following table.

| | Ex. | Base | Logarithm | Number |
|---|---|---|---|---|
| a) $3^4 = 81$ | a. | | | |
| b) $2^5 = 32$ | b. | | | |
| c) $5^2 = 25$ | c. | | | |
| d) $10^3 = 1000$ | d. | | | |
| e) $b^x = N$ | e. | | | |

3. T: 368 (1) - (3). Use the Table of Powers of 2 to explain the meaning of $2^8 = 256$. What other word would be correct for the word exponent in the table? Find the values asked for in exercises (1) to (3). If you finish before the answers are discussed, look up the following so that you may report to the class: R-1: 97:1; R-5: 367-368:8; 92:3.

4. Do you recall that ten was the base used by Briggs? Logarithms to the base ten are found in table form. They are referred to as tables of common logarithms. See pages 472 and 473 in the text. Suppose that we want to express 100 as 10 to a certain power. The table is not necessary because you know that $10^2 = 100$, so the logarithm is 2. Again, $10^3 = 1000$, so that logarithm is 3. However, the power of 10 which equals 7.23 cannot be found unless we use the table. Note that the 7.2 of 7.23 can be found on page 473 under the

column headed "No." and the 3 may be found in the upper row, also labeled "No." Slip a paper under 7.2 and the row of numbers to the right of 7.2. Read down the column headed by the number 3 until you reach the paper and you will find 8591. This is part of the missing logarithm. Think of it as the exponent. What was the base? Do you think $10^{8591} = 7.23$? What is the nearest you can come to 7.23 if you try to guess the nearest power of 10? How about $10^{2.8591}$, $10^{1.8591}$, $10^{0.8591}$? You note that 0.8591 is the nearest because $10^0 = 1$, $10^1 = 10$, and $10^{0.8591}$ belongs between $10^0$ and $10^1$, just as 7.23 belongs between 1 and 10. Note how they appear in column form:

$$10^{0.0000} = 1.00$$
$$10^{0.8591} = 7.23$$
$$10^{1.0000} = 10.00$$

For this next exercise disregard the decimal points on pages 472 and 473 as you will not need them. Express 526 as 10 to a certain power. Before you look at the table, decide whether you expect the result to start with $10^0$, $10^1$, $10^2$, $10^3$, or 10 to some other power. What did you decide? Note that this part of the logarithm does not appear in the table. You must find it by comparing 526 with the following possibilities: $10^0 = 1$, $10^1 = 10$, $10^2 = 100$, $10^3 = 1000$. You probably reasoned that 526 is greater than 100 and less than

1000, so that $10^2$ was the best that you could do. Now look for 52 and 6 in the margins called "No.". Near the bottom of page 472 you find 52, disregarding the decimal point, and by looking in the 6 column you find 7210 in the 52 row across. Now finish the $10^2$ by writing $526 = 10^{2.7210}$.

*4b

*4c

5. Form study groups of three or four pupils in a group and practice until you are sure you can express numbers as powers of 10. Next, see how to use this knowledge to multiply and divide numbers. See if you can figure out how to find the number which corresponds to $10^{2.8463}$. Remember that the 8463 is all that will appear in the table.

T: 369-375. Omit the rule in heavy print on 373.

R-2: 63-65. This gives clear directions for using the table. Try page 66:1-8.

R-3: 78-79. This is a little advanced, but look it up if you are interested in five-place tables.

R-4: 225-226. A good explanation of the decimal part of the logarithm.

R-5: 373. Explanation of the table. Try ex. 1-16. Omit the intervening pages and try page 377.

R-8: 92:4

R-9: 201-202. Use the table on page 201 and prepare to show the class how to use it to solve examples 1, 2, and 3 by logarithms.

Keep copies of the work of your group and appoint one member of each group to take part in a program of approximately thirty minutes. Each participant will have about five minutes in which to present to the class the most interesting or most difficult material selected by his group. Each group will also prepare one question and hand it to the teacher to be used in a quiz which will be given the day following the program. (Test 1)

6. In your reading, or in the program, you probably learned the names for the integral and decimal parts of the logarithm. If you did not, see page 374 in the text book.

II Predetermined Instruction Point (7)

7. There is another way to write $2^3 = 8$. Remember that 3 is the logarithm, 8 is the number and 2 is the base. The same fact may be expressed: The logarithm of 8, to the base 2, is 3. Note that if you omit the middle part, you are saying that the logarithm is 3. This is easy to remember if you recall that it amounts to saying that the exponent is 3. Try to express $5^2 = 25$ in the new form. What is the logarithm? It is the logarithm of what <u>number</u>? It is the logarithm to what base? The logarithm of 25, to the base 5, is 2. Try $10^{2.8756} = 751$

in the logarithmic form, which is the name of the new form. Can you see why the correct way is: The logarithm of 751, to the base 10, is 2.8756? Now abbreviate logarithm to log and write the three expressions as $\log_2 8 = 3$, $\log_5 25 = 2$, and $\log_{10} 751 = 2.8756$. Note where the base is written. Whenever the base is 10 it may be omitted. Thus, $\log 593 = 2.7767$ means $10^{2.7767} = 598$. The statement $5^4 = 625$ is called the exponential form and $\log 625 = 5$ is the logarithmic form. To master these two forms, form groups again and practice reading and writing the two forms interchangeably. See the following:

T: 375, 376:1-48. R-1: 97:4; R-5: 369:1-16; R-8: 92: 1-14. Note that you are expected to write $\log_2 8 = x$ as $2^x = 8$, and then see that $x = 3$. If you run into trouble in this set, such as $1/3 \log_3 27 = x$, consult the teacher or members of another group. R-9: 203:1-53.

A special test has been prepared on the above work with many questions based on the work in the text and reference books. You may take this test when you think you are ready for it. You may not go on to item 8 until you have passed this examination. (Test 2)

8. There is a short cut for finding the characteristic of a logarithm. See if you can discover it for yourself in the following way. Check the characteristics in the following table by comparing the integral part of

the number with integral powers of 10; for instance, 7 is between 1 and 10, or between $10^0$ and $10^1$, so the characteristic of log 7 is 0.

Table for Item 8

| Number | Logarithm |
|---|---|
| 7.3 | 0.8633 |
| 73.0 | 1.8633 |
| 730.0 | 2.8633 |
| 7300.0 | 3.8633 |
| 73000.0 | 4.8633 |
| 730000.0 | 5.8633 |

Do you see any relation between the characteristic and the number of digits in the integral part of the number? Check with the teacher on the wording of this short cut. It may be used from now on in determining the characteristic of the logarithm of a number. Review the following using the short cut. T: 372:10-19; R-5: 173:1-16: R-9: 209:1-9, 16-17. Keep this work.

9. Have you yet found the term antilogarithm? Be sure that you can use it. T:374. R-5: 371:1-12; R-9: 207: 1-24.

10. Suppose that log N = 3.7474. The mantissa .7474 as found in the table corresponds to the number 559. Since $10^3 = 1000$, $10^{3.7474}$ suggests 5590 as a more probable value of N. Briefly, if log N = 3.7474, then N = 5590.

11. Form groups again and practice finding the numbers whose logarithms are given. In other words, when the log of the number is given, find the antilogarithm. If you cannot find the mantissa in the table, use the nearest one to it. T: 376:36-48; R-4: 230. A good explanation. Read it if you are puzzled about this phase of the work; R-5: 375:1-9 (omit 5); R-9: 211. Another good explanation. (Ex. 1). You had probably better omit the explanation of ex. 2 at this stage of the work unless you are working ahead of most of the class. In any case, do ex. 1-6, 11-24. Keep a record of this work. Do you see a short cut for determining the number of significant digits in the number?

12. Using the results of items 8-11, study for a twenty minute quiz program to be given on a day chosen by the class. One group of pupils chosen the day before will prepare the questions and conduct the quiz, but before the quiz questions are given, this group must be ready to explain any difficulties raised by the rest of the class.

III Predetermined Instruction Point 13

13. We are now going to postpone any more technicalities to a later date and use logarithms to solve problems. Remember that the answers we shall obtain will be approximations only. The work will be similar to what you did in item 5, but we shall pay more attention to the form. Keep all this work in your notebooks and later we will add one more step and obtain a closer approximation by interpolating in finding the antilogarithm.

Find the area of a circle with radius 7.75", using $\pi = 3.14$.

$$A = \pi r^2$$

$$A = 3.14\ (7.75)^2$$

$$\text{Log } A = \log 3.14 + 2 \log 7.75$$

$$\text{Log } 3.14 = 0.$$

$$2 \log 7.75 = 2(0.\quad) = \underline{\qquad\qquad \text{Add}}$$

$$\text{Log } A =$$

$$A =$$

Note that in the plan, or set-up, the work is prepared and includes the characteristics. The table of mantissas is not used until the complete set-up is written. Before you complete the exercise above, consult T: 377, R-2: 65; R-5: 381; R-9: 213. Next, set up the following exercises with everyone in the class helping. Omit any exercises with numbers less than one. Do at least twenty exercises, some easy and as many hard ones as you can do.

14. T: 378:4, 9, 21, 22, 24, 28, 32.

15. Complete the circle problem and the others which have been set up. (Individual work)

16. Group work. Set up and solve any twenty of the following, referring to the answer book whenever you have completed at least five exercises. Try to do at least ten hard ones.

    T: 378:1-3, 5-8, 10-20, 23, 25-27, 29-31, 33-34.

    T: 381:31-46.

    T: 388:4,6,7.

    T: 389:2,5,7,8,9.

    R-1: 97:6, 12; 98:22, 25 (A good hard one. You are good if you can do it. If you get it right, put the completed copy on the bulletin board near the picture of Napier.)

    R-2: 69:4, 5, 8 (Swimming ponds, gasoline tanks, brass balls. Worth trying.)

    R-5: 382:1-12; 383:3, 6 (law of gravity, good for physics students), 8, 10 (two more for the bulletin board).

    R-6: Aviation problems: 12: 1, 2, 4-8.

    13: 1 Wright, Aeronca, Franklin, Lycoming. 105-113: Any of the design problems. When values are given to four significant digits, round them off to three before solving. 123+ 130: Round

off, as above. Can you evaluate $\frac{2\pi R\ \pi T}{360}$ by log's?

R-8: 93:1-3.

R-9: 212:1-5, 16-17, 19-20, 22 (Easy ones).

213:1-5, 15

215: Ex. 132: 1-6, 10-11, 14-15

215: Ex. 133: 1-12, 19, 22-25

216:1-2 Simple interest and surface of cylinder.

Use $\pi = 3.14$

4(a) Volume of cylinder

5(a)-(c) Volume of cone

217:8-14 Draw a geometric figure for each.

*16d

*16e

IV Predetermined Instruction Point 17

17. So far, you have not worked with numbers less than 1. Suppose that you plan to multiply 746 by .943. Log 746= 2.8727 but what is log .943? The mantissa is 9745, but what is the characteristic? Can it be 0? Less than 0? What numbers are less than 0? Try -1. What is $10^{-1}$? Check the table below.

| Powers of 10 | Numbers | N | Characteristic of Log N | Mantissa of Log N |
|---|---|---|---|---|
| $10^{-3} = \frac{1}{10^3}$ | .001 | .00943 | -3 | .9745 |
| $10^{-2} = \frac{1}{10^2}$ | .010 | .0943 | -2 | .9745 |
| $10^{-1} = \frac{1}{10}$ | .100 | .943 | -1 | .9745 |
| $10^{0}$ | 1.000 | 9.43 | 0 | .9745 |
| $10^{1}$ | 10.00 | 94.3 | 1 | .9745 |
| $10^{2}$ | 100.00 | 943. | 2 | .9745 |
| $10^{3}$ | 1000.00 | 9430 | 3 | .9745 |

First refer to the preceding table for help in determining the characteristics of the logs of the following. Find the mantissas as usual. See how soon you can find the characteristics without referring to the powers of 10. There is a short cut rule here, too, which you may use as soon as you catch on to it. Keep the characteristics and mantissas in separate columns as shown. The reason for this will be apparent later. T: 381:1-9; R-5: 371:5, 7-12; R-9: 207:11-24.

| Number | Characteristic | Mantissa |
|---|---|---|

18. Fill in the following blanks:

In the work in item 5, you multiplied numbers by _____ the logarithms of the numbers, you divided numbers by _____ the log of the divisor _____ the log of the dividend, you raised numbers to powers by _____ the log of the number ____the power, and you found roots of numbers by _____ the log of the number ____ the index, in all cases finishing by finding the antilogarithm.

19. The numbers with which you worked were greater than 1. To use numbers between 0 and 1, there is a way of writing negative characteristics which make them easier to handle. The table in item 13 and your answers to item 14 show that while the characteristics of decimals are negative, the mantissas are positive. For the new way of expressing negative characteristics read the following pages and try the exercises found there:

T: 379-381: 1-18, 31-34.

R-2: 66 (Read)

R-5: 379:1-4

R-9: 206-207:11-23; 209:10-20

V Predetermined Instruction Point 20

20. Ex. 30 on page 378 in the text resulted in log N = 0.3013. You probably decided that N = 2.00 because the mantissa 3010, which was in the table, was the best you could

do for 3013. By a process called interpolation, it is possible to come nearer to the number whose logarithm is 0.3013. The mantissa 3010 corresponds to 2000 and the next mantissa in the table 3032 corresponds to 2010, so the mantissa 3013 corresponds to a number greater than 2000 and less than 2010. To find how far beyond 2000 it is, we make a little table as follows:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| mantissa | 3010 | corresponds | to the | number | | 2000 | Leave blank at first. |
| " | 3013 | " | " | " | " | | |
| " | 3032 | " | " | " | " | 2010 | |

The difference between 3010 and 3013 is 3. The difference between the book mantissas 3010 and 3032 is 22. This is called the tabular difference, since it is the difference between the mantissas that are in the tables. The mantissa which is not in the table, 3013, is $\frac{3}{22}$ of the difference between the mantissas in the table. Thus, we expect the missing number to be $\frac{3}{22}$ of the difference between the two numbers in the table. That difference is 10, so we find $\frac{3}{22}$ of 10, which is approximately 1.3, or for our purposes 1. Add this to the 2000, making 2001. Now see the characteristic. The value N = 2.001, to four significant digits. The work looks like this:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Mantissa | 3010 | corresponds | to the | number | | 2000 | (The last zero is added to the number obtained from the table just to keep us working with four-digit numbers) |
| " | 3013 | " | " | " | " | | |
| " | 3032 | " | " | " | " | 2010 | |

$\frac{3}{22} \times 10 = \frac{30}{22} = 1.3$ or 1

2000 + 1 = 2001 (Insert it in the missing place in the table.)

The best way to learn to interpolate is to do it under guidance, so we will take any values of log N that you have as a result of item 20 and do them together. As soon as you think you have learned the process, you may be the leader.

Warning: Diferent books give different ways of writing the interpolation, so here is one place where you had better not refer to the books. It will be much clearer if everyone uses the same form. If you wish you may soon omit the word <u>number</u> and write:

| | | | | | |
|---|---|---|---|---|---|
| Log N = 2.8467 | Mantissa | 3463 | corr. | to | 7020 |
| N = 702.6 | " | 8467 | " | " | 7026 |
| | " | 8470 | " | " | 7030 |

4/7 x 10 = 40/7 = 5.7 or 6

7020 + 6 = 7026

21. When you have practiced on interpolating, it will be time to take Test 3.

*21f

22. Go back to item 13 and try twenty-five harder exercises, interpolating at the end. You may work in groups of four or less, and apportion problems in the reference books to members of the group. Each group will have two or three days to prepare for a program to consist

of the two most difficult exercises done by that group. Sign up for them on the program on the bulletin board as follows:

1) Name of Pupil R-6: 12:8.

2) Name of Pupil R-9: 216:8.

Are you having trouble with such steps as

0.7634 or 3/7.1644-10 ?
9.1847-10 (subtract)

If so, consult T:380; R-5: 379,380; R-9: 213. Read the explanation in fine print. If you are still puzzled, arrange for a conference.

VI Predetermined Instruction Point 23.

23. Up to this stage, all exercises dealt with numbers of three significant figures. To find logs of numbers of four significant digits, interpolate as you have been doing in finding antilogarithms. To find log 275.5, write the characteristic first, as you have been doing. Log 275.5 = 2 _____. Now interpolate, starting in the number margins, looking for the nearest numbers to 2756. Omit the decimal point since the characteristic has been found. You can find log 2750 and log 2760. Arrange them as follows:

mantissa 2750 = 4393

2755 = (4401) (Leave vacant at first, and fill in after the interpolating has been completed below.)

2760 = 4409

Although the Number margins contain only 275 and

276, you add the zeros above to keep four digits. Now the work is the same as in the other interpolating. 5/10 x 16 = 8. Add 8 to 4393. The missing mantissa is 4393 + 8 or 4401, so log 275.5 = 2.4401.

Did you notice that 2755 was halfway between 2750 and 2760? You found 1/2 the difference between 4393 and 4409 and added it to 4393. Some pupils prefer to use .5 instead of 1/2 as they find it easier to round off the product.

24. Here is another illustration of interpolation in finding the logarithm of a number. Become familiar with the procedure, as it will be easier if all pupils follow the same routine.

Required to find log 3.142.

Log 3.142 = 0.4972 Mantissa 3140 = 4969

" 3142 = 4972

" 3150 = 4983

14

.2

2.8 or 3 4969 + 3 = 4972

Compare the work above with the table on page 472 of the text book.

25. Practice interpolating, then take Test 4. Mark the following practice material and be able to do at least twenty without an error before you attempt the test. T: 387:1-12, 25-49.

R-4: See 232 for log $\pi$ when a five-place table is used.

R-5: 374:1-25; R-9: 210:1-20.

26. See page 383 in the text or R-9: 208 for the graph of logarithmic relation, y = log x. Complete the following statements by consulting the graph:

a. _____ numbers do not have common logarithms.

b. The logarithm of a number greater than 1 is _____.

c. The logarithm of a number between ____ and ____ is negative.

d. As a number increases, its logarithm _____.

e. When x increases from 3 to 4, log x increases .14.
When x " " 4 to 5, log x " .10.
(See the table as well as the graph.)

f. The increase in the logarithm is not _____ to the increase in the number.

g. This does not interfere with the method of interpolation, however. Why not? See T: 383 or R-9: 208 (Fine print at bottom of page).

h. What does the graph indicate concerning log 0?

*26g

27. Before solving more problems by logarithms, it is a good idea to state the procedures we have used in multiplying, dividing, raising to powers, and finding roots.

Log ab =

Log a/b =

Log $a^m$ =

Log $\sqrt[m]{a}$ = log $a^{\frac{1}{m}}$ =

28. To test your understanding of logarithms, take the True-False Test on page 382 of the text. A similar test will be given (Test 5) when you have mastered the work on 382.

29. Before solving more complicated problems, review the set-up, item 17. Pay especial attention to radical signs. The following set-up illustrates the problem: Find the radius of the largest gas tank that could be placed on a triangular plot 77.53 feet by 98.47 feet by 84.03 feet.

$r = \frac{K}{s}$ where r is the radius of the inscribed circle, K is the area of the triangle and s is half the perimeter. Since $K = \sqrt{s(s-a)(s-b)(s-c)}$, the formula may be expressed as:

$$r = \frac{\sqrt{s(s-a)(s-b)(s-c)}}{s}$$

$$r = \frac{\sqrt{130.0(52.49)(31.55)(45.99)}}{130.0}$$

Log r = $\frac{1}{2}$(log 130.0+log 52.49+log 31.55+log 45.99)-log 130.0

log 130.0 = 2.

log 52.49 = 1.

log 31.55 = 1.

log 45.99 = 1. (Add)

$\frac{1}{2}$( )

=

log 130.0 = 2. _____ (subtract)

log r =

r =

Study the following set-ups: T: 377 and 388; R-4: 233; R-5: 381. R-9: 213 and 214. Do you notice a step in the sample above and in R-9 that is missing from the other illustrations? It is a good idea to put it in.

30. When an exercise involving negative numbers is to be solved by logarithms, it is essential to remember that negative numbers do not have logarithms. To get around this difficulty, see T: 387.

*30h.

31. Now comes a chance to solve exercises which arise in arithmetic, in algebra, in geometry, in chemistry, in physics, in aviation, and in electricity. Try your skill in all fields. Do correctly at least five of each type before you try the optional related work. Keep a notebook of your work.

A. General type - arithmetic and algebra:

T: 387:25-53
388:1-16

R-1: 97:6-17
98:18

R-2: 66:1-8
69:1-2

R-4: 85:1-20 (Round off to four significant digits and use four-place tables.)

R-5: 382:1-8

R-7: 40:5-7, 15

R-8: 93:1-10

R-9: 215:1-31
216:1(a)-(d)
217:15(a)-(b)

B. Geometry:

T: 389:1-2, 5, 7.

R-1: 98:19-22

R-2: 69:4, 5, 6, 8

R-5: 383:1-3, 8-10
388:101-105.

R-9: 216:2, 4, 5.
217:8-14

C. Chemistry:

T: 389:3 Bring in further illustrations from your chemistry classes.

D. Physics:

T: 389:4

R-1: 98:24

R-2: 69:7

R-5: 383:4-7

R-9: 216:3, 6

E. Aviation:

R-6: Any suitable formulas. See pages 12-13, 105-113, 123-130.

F. Electricity:

R-1: 98:23. Look through the Westinghouse Electric Notes
R-9: 217:16. and Data Book and choose appropriate exercises.

*31i.

32. Pick out the most interesting or the most difficult problem in your notebook and hand it to the committee chairman. Be prepared to explain it if it is chosen for the bulletin board or for a program.

VII Predetermined Instruction Point 33.

33. Even though addition and subraction cannot be done by logarithms, certain parts of formulas involving addition and subtraction can be done by logarithms. If the hypotenuse of a right triangle is to be found, logarithms can facilitate such work.

$$c = \sqrt{a^2 + b^2}$$

$$= \sqrt{3.75^2 + 5.88^2}$$

Let $x = 3.75^2$ and $y = 5.88^2$

$\text{Log } x = 2 \log 3.75$ and $\log y = 2 \log 5.88$

$= 2(0.5740)$ $\quad = 2(0.7694)$

$= 1.1480$ $\quad = 1.5388$

$x = 14.06$ $\quad y = 34.58$

$$c = \sqrt{14.06 + 34.58}$$

$$= \sqrt{48.64}$$

$$\text{Log } c = \tfrac{1}{2} \log 48.64$$

$$= \tfrac{1}{2}(1.6870)$$

$$= 0.8435$$

$$c = 6.972$$

$$\therefore c = 6.97$$

34. How would you solve the right triangle formula $b = \sqrt{c^2 - a^2}$ if c = 179.6 and a = 121.5?

Solve the following right triangle exercises if c is the hypotenuse and a and b are the arms, and K = area.

1) a = 19.4, b = 23.8, c =

2) a = 927, b = 746, c =

3) a = 12.5, c = 23.7, b =
See R-9: 217:17-18

4) K = 729.6, b = 297.4, a =

5) K = 27.6, a = 8.5, b =

6) K = 2345, c = 149, find the altitude to the hypotenuse.

35. Prepare a copy of a page of a savings bank book. Make the correct entries for the following facts: Deposit of $5000 on Dec. 1, 1940; June 1, 1941, compound interest at 1½%; Dec. 1, 1941, interest compounded at 1½%. (Compute the interest on the even dollars. In adding interest to the previous balance, ignore any fractional part of a cent). Continue as before for June 1 and Dec. 1, ending with Dec. 1, 1946. What is the amount of money in this account? What is the interest? How would you like to find the amount compounded semi-annually for 10 years?

Banks use interest tables, which we do not have available. However, we can solve compound interest problems by logarithms, to find not only the amount, A, but to find the <u>present value of A</u>, that is, to find the principal which must be invested under certain conditions to amount to any given value of A. What

principal, invested at 3% interest compounded semiannually, when you were one year old, would amount to $5000 when you were eighteen years old?

T: 390
391:1-12

R-1: 98:26
99:27

R-5: 415:1-4
417:3-4 Note that the solution here is based on a new topic, geometric series, but that logarithms are necessary, too.

R-5: 432:2-11

R-8: 93:11-13

R-9: 216:7(a)-(f) Study 218 for the derivation of the compound interest formula. Solve at least ten exercises. 219:1-21.

*35j.

36. In how many years will $350 amount to $1000 at 4% compounded annually? Set it up by the formula $A = p(1+\frac{r}{k})^{kn}$. Now note that the unknown is the exponent. Consult T: 392:1-18 R-9: 220:1-17.
393:19-22.

37. In preparation for the final test, take one of the following, allowing one full period for it:

T:399:1-13 and 400:14-18; Part II, 400: 1-8
R-4: 309:Ch. 30 (1)-(5).
R-5: 384:A 1-13
R-9: 221:1-15

*371.

38. For the last discussion period before the final test, bring in any questions which have caused you

difficulty. If you have none, try the following:

R-1: 144:8

R-2: 85; 64; 90:111, 112, 113, 114, 115; 103:3.

R-4: 314:II (2)

R-5: 382:Exer. C, 6

R-6: 460:4

R-7: 40:13

R-8: 95:1-7, 9-10; 96:11, 19-26

R-9: 221:7, 9(c), 11(e); 222:12; 254:7(a)-(f); 260: 5, 8, 9, 11

*38m.

## Pupils' Bibliography for Core Activities

### Textbook

Schorling, Raleigh; Smith, Rolland R.; and Clark, John R., Second-Year Algebra, Schorling-Clark-Smith Mathematics Series, World Book Company, Yonkers-On-Hudson, New York, 1942, 484 pages.

### Reference Books

1. Bartlett, Josiah; Creelman, George W.; Ewing, Cecil A.; McCormick, Islay F.: Rich, Ernest E.; Wilson, George R., Algebra Review Exercises, Ginn and Company, Boston, 1928, 151 pages.

2. Buchan, A. F.; Barthwick, R.; Wadden, William R., Aviation Mathematics, Houghton Mifflin Company, Boston, 1942, 136 pages.

3. Curtiss, David Raymond; Moulton, Elton James, Essentials of Trigonometry with Applications, D. C. Heath and Company, Boston, 1942, 94 pages.

4. Hooper, A., A Mathematics Refresher, American Edition, Henry Holt and Company, New York, 1942, 342 pages.

5. Lennes, N. J., A Second Course in Algebra, Revised Edition, The Macmillan Company, New York, 1943, 522 pages.

6. Osteyee, George, Mathematics in Aviation, Air-Age Education Series, The Macmillan Company, New York, 1942, 186 pages.

7. Smith, David P., Jr.; Fagan, Leslie T., Mathematics Review Exercises, Ginn and Company, Boston, 1940, 280 pages.

8. Tower, Oswald, and Sides, Winfield M., Reviews and Examinations in Algebra, D. C. Heath and Company, Boston, 1943, 298 pages.

Teacher's Bibliography

Bell, Eric Temple, The Development of Mathematics, McGraw-Hill Book Company, Incorporated, New York, 1940, 583 pages.

Billett, Roy O., Fundamentals of Secondary-School Teaching, Houghton Mifflin Company, Boston, 1940, 671 pages.

Breslich, E. R., Problems in Teaching Secondary-School Mathematics, The University of Chicago Press, Chicago, 1931.

_____, The Technique of Teaching Secondary-School Mathematics, The University of Chicago Press, Chicago, 1930.

Cajori, Florian, "Talk on Logarithms and Slide Rule," School Science and Mathematics (June 1920) 20:527-530.

Hassler, J. O. and R. R. Smith, The Teaching of Secondary-School Mathematics, The Macmillan Company, New York, 1930.

Lide, Edwin S., Instruction in Mathematics, Bulletin, 1932, No. 17, National Survey of Secondary Education, Monograph No. 23, United States Office of Education, Washington, D. C.

Multi-sensory Aids, Eighteenth Yearbook, National Council of Teachers of Mathematics, Bureau of Publications, Teachers College, Columbia University, New York.

Smith, David Eugene, A Source Book in Mathematics, McGraw-Hill Book Company, Incorporated, New York, 1929, 697 pages.

The Place of Mathematics in Secondary Education, Fifteenth Yearbook, National Council of Teachers of Mathematics, Bureau of Publications, Teachers College, Columbia University, New York.

Wilt, Mary L., "An 'Overall' Method of Teaching Logarithms," The Mathematics Teacher (March 1947), Volume 40, No.3: pp. 133-135. (This article appeared after the completion of this unit.)

Woodring, Maxie Nave and Vera Sanford, Enriched Teaching of Mathematics in the Junior and Senior High School, Bureau of Publications, Teachers College, Columbia University, New York, 1938.

## Directions for Optional Related Activites

The directions as given here were copied from the directions on the library cards to which the pupils were referred in the pupil study guide.

### Optional Related Activities

*1a. Biographical information concerning John Napier. Where born? When? Family and background. Education. Published works. Two other great interest beside mathematics in his life. Names of two of his friends. What great interest did they have in common? How did one of them add greatly to the value of Napier's work in logarithms? Human interest stories.

The Columbia Encyclopedia - See Napier, John.

The Encyclopedia Americana - See Logarithm and Napier.

The Encyclopedia Brittanica. - R to R9 in the special bibliography and any other sources that occur to you. Prepare a coherent, attractive report suitable for the bulletin board.

History of Mathematics, Volume I, David Eugene Smith. See page 389 for a picture of Napier and pages 389-390 for information on Napier, and page 391 for Briggs.

*4b. The use of the Base e.

The Columbia Encyclopedia - See logarithms. Part of this is very technical, but try it to see how much of

it you can follow. You should at least know that there is such a base used in higher mathematics.

The Encyclopedia Brittanica. See page 301, from the last two lines on the left to the end of the first paragraph on the right of the page.

Write a brief summary, one or two sentences only if you wish, and put it on the bulletin board.

Remember that R-1 to R-9 in the special bibliography may contain references to the base e.

History of Mathematics, Vol. II, David Eugene Smith. See the first paragraph on page 517.

* 4c. Other tables: Fiske and Longley; Huntington; Hunn and MacInnes. Tables you may have at home.

It is a good idea to become acquainted with logarithmic tables arranged in different ways.

When you are familiar with the tables in the text and in the reference books, which are very much alike, use other tables for item 5, 9, and succeeding items.

Ask the teacher for any of the following:

1. Fiske and Longley, Four-Place Mathematical Tables.
2. Hun and MacInnes, Logarithmic, Trigonometric and other Tables (five-place tables)
3. Huntington, Four Place Tables of Logarithms and Trigonometric Functions.

If your father or brother has a table of logarithms arranged differently, ask if you may borrow it to show to your classmates.

*16d. Electricity Problems. Westinghouse Electric Notes and Data Book.

Look through the formulas and list ten or more which lend themselves to logarithmic solution. Label them according to their use and identify the variables. Many of them require more mathematics than the class has had, but many more can be solved now. From the tables choose values, solve the equations, and check your results by reference to the table. Prepare a neat report of your work.

Cooke, Mathematics for Electricians and Radiomen.

Take good care of this book. It has been lent by the father of one of the pupils. See pages 280-308 and the table on 509.

Try the electrical problems on pages 304-308 if you want a good work-out. There is an answer book attached. Use it wisely. Perhaps you can obtain a book of this type that we may borrow for the duration of the unit.

*16e. The Cologarithm.

It is possible to solve problems by logarithms without ever using the cologarithm of a number, but the use

of the colog is a convenience.

Twelfth-grade pupils make frequent use of cologs, so you may like to learn to use them now.

Robbins, Plane Trigonometry, pp. 132-134. Note the answers in the back. Try some of the exercises on pages 133-134.

Look up Cologarithm in the index of the other reference books or in the encyclopedias in the library.

Make a neat copy of several exercises in which you used cologarithms, and put it on the bulletin board.

From now on you may use cologarithms in the solution of exercises in the core material.

*21f. Proportional Parts Tables.

Hun and MacInnes, Logarithmic, Trigonometric, and other Tables, pp. 197-199. See how the table of proportional parts simplifies the interpolating in five-place tables.

Huntington, Four Place Tables of Logarithms and Trigonometric Functions. Practice using the table of "Tenths of Tabular Difference."

See the article on logarithms in Volume XIV of the New International Encyclopedia in the school library for a reference to logarithms to fourteen decimal places. Take notes for a class report.

*29g. Logarithmic Curve.

See page 383 in the text or look up the logarithmic curve in any of the references.

Van Nostrand's Scientific Encyclopedia in the school library contains a good article on the curve.

Study the directions on page 383 or in other references and make a large copy of the graph y = log x for class use. List on your copy five important facts demonstrated by the graph.

*30h. C.E.E.B. Exercises.

Ask the teacher for College Entrance Examination Board Examination Questions in Mathematics.

If you note the page and number of the exercise as follows: 27:15, the teacher will be able to correct your work.

12:9

21:20

27:15

If you have a pamphlet, record your work as:

June 1938: 2:9.

7:6.

*31i. Problems from Outside Sources, Other Departments of the School, Parents' Employment, Student Reading, or Examination Questions.

Start a collection of problems in which logarithms are used, but which do not occur in mathematics textbooks. (Class Scrapbook)

Suggestions:

Chemistry: Prepare a report on pH. The chemistry teacher has offered to help chemistry students with this.

Physics and Aeronautics: If you take these courses, prepare a report demonstrating the logarithmic solution of formulas which you have used.

Use in Vocations: See if a member of your family or a friend uses logarithms for business, industry, or a profession. Ask them for an illustration of such usage. Do you know any astronomers, physicists, or other scientists?

College Courses: Ask your friends who are in college to give you information regarding courses in which they use logarithms. They may give you a copy of their examinations which contain such questions. See if you can solve them.

*35j. Compound Interest Accumulation in a Savings Bank Book.

Arrange a sheet of oak tag to resemble a page in a savings bank book. Copy one of your own or use the following:

| Date | Deposit | Withdrawal | Dividend | Balance |
|---|---|---|---|---|

Use either semiannual or quarterly dividend periods, April and October dates, or April, July, October,

January dates, or any others that you find in a real bank book. Time yourself after you have begun the actual figuring.

Make entries in the proper columns after you have chosen the amount to deposit. (r = 3 per cent figured semiannually.)

| Date | Deposit | Withdrawal | Dividend | Balance |
|---|---|---|---|---|
| 1947 | | | | |
| Mar.20 | $500.00 | | | $500.00 |
| Oct.15 | | | $7.50 | 507.50 |
| 1948 | | | | |
| Apr.15 | | | 7.60 | 515.10 |
| Oct.15 | | | 7.62 | 522.82 |

Do not make withdrawals, and disregard anything less than a dollar when figuring the interest. Find the balance as of March 1957. Now solve the same exercise by logarithms and compare the time taken by the two methods.

Did your two final balances agree to the nearest cent, ten cents, dollar or ten dollars? How do you account for this?

*37k. References to Twelfth Grade Textbook.

Here are some exercises in Plane Trigonometry, Robbins, which require a knowledge of logarithms but not of trigonometry. Try any exercises on pages 127 to 140. These pages contain a good review of this unit and a little advanced theory of logarithms. Page 135, exercise 17, is a challenge to the best students.

Try it and choose others of a similar type.

Other challenging ones are 143:1, 2; 144:21; 145:32; 147:79; 148:89, 94, 95; 149:110; 150:115. 150:117 is an interesting one which is also easy.

*371. Practice reading aloud "The Wennelsprocket Test of Reading Ability, or, Torture in 2 Paragraphs," in The Clearing House, January 1947, Volume 21, Number 5, page 267.

*38m. Collection of References to Logarithms in General Reading.

If you come across any reference to logarithms in newspapers, magazines, or any other readings, clip it out or copy it with a notation of the source, and bring it in for the class collection.

Bibliography for Optional Related Activities

College Entrance Examination Board Examination Questions in Mathematics, Sixth Series, 1926-1930, Ginn and Company, Boston, 1926, 123 pages.

_____, Pamphlets, 1927-1938, Ginn and Company, Boston, 1927-1938.

Cooke, Nelson N., Mathematics for Electricians and Radiomen, McGraw-Hill Book Company, Incorporated, New York, 1942, 549 pages.

Fiske, Thomas Scott and Longley, William Raymond, Four-Place Mathematical Tables, The Macmillan Company, New York, 41-235, 205 pages.

Huntington, E. V., Four-Place Tables, Unabridged Edition, Houghton Mifflin Company, Boston, 1910, 32 pages.

Robbins, Edward R., Plane Trigonometry, American Book Company, Boston, 1909, 154 pages.

Ryan, H. H., "The Wennelsprocket Test of Reading Ability, or, Torture in Two Paragraphs", The Clearing House, (January, 1947) 21: 5:p. 267.

Smith, David Eugene, History of Mathematics, Volume I, Ginn and Company, Boston, 1923, 596 pages.

_____, History of Mathematics, Volume II, Ginn and Company, Boston, 1925, 725 pages.

_____, Portraits of Eminent Mathematicians with Biographical Sketches, V, Portfolio I, 1946 Edition, Scripta Mathematica, New York, 1946.

The Columbia Encyclopedia, Ansley, Clark F., Editor-in-Chief, Columbia University Press, Morningside Heights, New York, 1935.

The Encyclopedia Americana, Americana Corporation, New York, 1938.

The Encyclopedia Brittanica, Fourteenth Edition, The Encyclopedia Brittanica, Inc., New York, 1938.

Van Nostrand Scientific Encyclopedia, D. Van Nostrand Company, Inc., New York, 1928.

# Logarithm Examination

Foreword: This is a pre-test of a new unit of work. The purpose of the test is to give you a preview of the new work and to give the teacher an indication of what to emphasize in the days to come.

Do your best work. If you are reasonably sure of a question, answer it. However, do not worry if you cannot answer some of the questions, and do not try to invent ways of doing exercises which are absolutely unfamiliar to you.

Your score will not be counted towards your mark in mathematics. Its chief value is to indicate a starting point in the new work.

## Part I

Directions: Read these statements and mark each one on the line to the right of the statement with a plus sign, +, if you think it is correct, or a minus sign, −, if you think it is incorrect in any way. Do any necessary computation in the left margin or on the back of the paper. Do not refer to books or tables for this part of the examination.

Samples: Logarithms were invented before 1620. __+__

Log 2 + log 5 = log 7. __−__

1. Logarithms are time-saving devices for certain mathematical processes. 1____

20. If the characteristic of a logarithm to the base ten is one, the number is between ten and one hundred. 20____

21. If a number is between one and ten, the characteristic of its logarithm to the base ten is one. 21____

22. It is not necessary to write the base when using the logarithmic form. 22____

23. If $\log_8 64 = x$, then $x^8 = 64$. 23____

24. $7^3 = 343$ may be expressed as $\log_7 3 = 343$. 24____

25. Log 13 = 1.1139 means the same as $\log_{10} 13 =$ 1.1139. 25____

26. If log N = 3.4010, N is a number between 10 and 100. 26____

27. The characteristic of log 41560 is 5. 27____

28. If log 45.67 = 1.6797, then log 456.7 = 16.597. 28____

29. Log 12 may be found without a table if log 2 and log 3 are known. 29____

30. Log 20 = log 50 + log 2 - log 5. 30____

31. If $\log_x 25 = 2$, then x = 5. 31____

32. If log 5.324 = 0.7262, then $532.4 = 10^{2.7262}$. 32____

33. The characteristic of the log of a number between zero and one is negative. 33____

34. The characteristic of log .0215 is negative one. 34____

35. Log .001 = 7.000 - 10. 35____

36. If log 48.54 - 1.6861, then log 0.04854 = 8.6861 - 10. 36____

37. The characteristic of a logarithm may be either positive or negative. 37_____

38. $\text{Log}_{10}(-1000) = -3$. 38_____

39. If the antilogarithm of 0.3010 is 2, then the antilogarithm of 9.3010 - 10 is .2. 39_____

40. If log x is negative, then x is negative. 40_____

41. If log 2 = 0.30103 and log 3 = 0.47712, then log .6 = 9.77815 - 10. 41_____

42. Log 0.0001 = -4. 42_____

43. If log N = 2.3680 and N = 233.3, then when log N = 8.3680 - 10, N = .02333. 43_____

44. Log N = 22.9538 - 20 means that log N = 2.9538. 44_____

45. If log 48.54 = 1.6861, log 4854 = 2.6861. 45_____

46. If log 75.70 = 1.8791 and log 75.80 = 1.8797, then log 75.78 = 1.8795. 46_____

47. If log N = 3.8741 and if the antilogarithm of 3.8739 is 7480 and the antilogarithm of 3.8745 is 7490, then N = 7483. 47_____

48. Log $\sqrt{10} = \frac{1}{2}$ log 10.

49. If a = 276 and b = 345, $\log a^2 + \log b^2 =$ 2 log 276 + 2 log 345. 49_____

50. If log 130.0 = 2.1139, then $\log \sqrt{130.0} = \sqrt{2.1139}$. 50_____

51. $\text{Log} \dfrac{a^2}{b^3} = \dfrac{2 \log a}{3 \log b}$ 51_____

52. Log $\frac{A}{N}$ = log A + log N. 52_____

53. If log a = 9.6231 - 10, then 1/3 log a = 3.2077 -10. 53_____

54. If log 3.5 = 0.5441 and log 7.12 = 0.8525, then log (350 x 71.2) = 1.3966. 54_____

55. $\text{Log}_3 1/3 = -1$. 55_____

56. $\text{Log}_a M^n = n \log_a M$. 56_____

57. Log $\frac{x}{y} = \frac{1}{y}$ log x. 57_____

58. Log $\frac{\sqrt{ab}}{c} = \frac{1}{2}$ (log a + log b - log c). 58_____

59. Log $a^4b$ = 4 log a (log b). 59_____

60. $\text{Log}_{10} 1 = 1$. 60_____

61. Log a - log b = log $\frac{a}{b}$. 61_____

62. If log 9 = 0.9542, log 81 = 1.9084. 62_____

63. If log 40 = 1.6021, log 20 = 0.8012. 63_____

64. $\text{Log}_5 625 = x$; x = 4. 64_____

65. $\frac{\text{Log } 10 + \log 2}{2} = \sqrt{20}$ 65_____

66. Log $\sqrt[3]{.002}$ = 1/3 log .002. 66_____

67. 1/3 (7.3010 - 10) = 1/3 (27.3010 - 30). 67_____

68. Given log a = 6.3283 - 10, log b = 7.7642 - 10, and log c = 9.1048 - 10; log abc = 3.1973 - 10. 68_____

69. 5(8.7505 - 10) = 3.7525 - 10. 69_____

70. If log x = 1.5678 and log y = 4.1568, then log $\frac{x}{y}$ = 2.5890. 70_____

71. $\frac{276 \times 1.99}{3} = \log 276 + \log 1.99 - \log 3.$ 71_____

72. Since $\sqrt[m]{10^n} = 10^{\frac{n}{m}} = \log 10^{\frac{n}{m}} = \frac{n}{m} \log 10 = \frac{n}{m}$. 72_____

73. $\text{Log } a - 2 \log b + \frac{1}{2} \log c = \log \frac{a\sqrt{c}}{b^2}$. 73_____

74. $1/3 \log_2 64 = 2.$ 74_____

75. Given $\log x = 4.1073$ and $\log y = 6.9860$;

$\log \frac{x}{y} = 7.1213 - 10.$ 75_____

76. When $s = \frac{1}{2}gt^2$, $\log s = \log g + 2 \log t - \log 2.$ 76_____

77. When $s = vt + \frac{1}{2}gt^2$, $\log s = \log v + \log t +$

$\log g + \log t - \log 2.$ 77_____

78. If $\log_{10} 40 + \log_{10} \frac{5}{2} = x$, $x = 2.$ 78_____

79. As a single logarithm, $\log_{10} x + 3 \log_{10} y - 2/3$

$\log_{10} Z = \log \frac{xy^3}{\sqrt[3]{Z^2}}.$ 79_____

80. If $V_2 = V_1 \sqrt{\frac{s_1}{s_2}}$, then $\log V_2 = \log V_1 + \frac{1}{2} \log s_1 -$

$\log s_2.$ 80_____

81. $\text{Log}_9 27 + \log_{10} 1000 + \log_5 625 + \log_2 1 = 10.$ 81_____

82. If $x = \log_{\frac{1}{3}} \sqrt[5]{27}$, $x = -1.4.$ 82_____

83. If $\log 2 = 0.3010$, there will be nine digits

in $2^{32}$. 83_____

84. If $\log_5 2x = 2$, $x = 12\frac{1}{2}.$ 84_____

85. If $27.4^x = 54.8$, $x = \frac{\log 54.8}{\log 27.4} = \log 2.$ 85_____

86. If $4^{x-1} = 256$, $x = \frac{\log 256}{\log 4} + 1.$ 86_____

87. If $\log x$ and $\log y$ have the same mantissas,

then $x = y.$ 87_____

# Logarithm Examination

## Part II

Directions: Write the result on the line to the right of the exercise. Do the computation on the back of the paper or in the upper and lower margins, except where space is provided.

91. Log 32.7 = 91_____

92. Log .003 = 92_____

93. If log N = 2.9736, N = 93_____

94. If log N = 9.6609 - 10, N = 94_____

95. Log 1.467 = 95_____

96. Log 3.292 = 96_____

97. If log r = 1.7474, r = 97_____

98. Log (540 x 72.3) = 98_____

99. If $x = \frac{74.5 \times 2.78}{1.26}$, complete the following set-up: log x = log 74.5 _____ 99

100-102. Refer to previous results if you wish to for the following three exercises. Give numerical results.

100. Log 4.36 - log 32.7 = 100_____

101. One-third log .003 = 101_____

102. Log 1.476 - $\frac{1}{2}$ log 269.5 = 102_____

103. How many digits are there in $17^{32}$? 103_____

104. Find $\sqrt[5]{88}$ by logarithms. 104_____

105. Log .05672 = 105_____

106. Find $\sqrt[3]{.05672}$ 106_____

107. By logarithms, solve $\frac{26.42 \times 269.5}{9.472}$ 107_____

108-111. There are four omissions in the following set-up. Write the correct responses in the proper sequence on the lines at the right.

$N = \sqrt{\frac{19.87(600.5)^2}{4827}}$ 108_____

Log N = (log 19.87 + log 600.5 ). 109_____

110_____

111_____

112-114. Show all the work for exercise 112, including the vertical set-up, in the space below. When you have found log x, write it on the first line. Write the value of x on the second line and leave the third one vacant for the teacher to score the neatness of the set-up.

$x = 300\sqrt{\frac{6249 \times 300}{.5295}}$ 112_____

113_____

114_____

115. The side of an equilateral triangle is 47.7.

Find the area by logarithms. 115_____

116-118. The formula for the heat in calories developed by an electric current i flowing through a conductor having a resistance r during a time t is $H = \frac{r\,i^2 t}{4.18}$.

Find H when r = 150, i = 0.75, and t = 45. 116_____

Follow the directions for exercises 112-114. 117_____

118_____

119. Find the present value of $5000 to be used in 18 years to send a boy to college, interest being compounded at three per cent semiannually. 119_____

120. Find x if $x^{\log_{10} x} = 100$. 120_____

Test 1

Directions: Do not copy the test items. Write the responses to the completion items and the true-false items in the column to the right. Do the necessary computation neatly in the upper and lower margins or on the back of the page.

1. The inventor of logarithms was _____. 1_____

2. The base 10 was introduced by _____. 2_____

Mark T if true and F if false:

3. Any number may be used as base in a system of logarithms. 3_____

4. The year ??14 is associated with the publication of a work on logarithms. 4_____

5-6. Two processes of mathematics not suitable for logarithmic solution are: 5_____ 6_____

7, 8, 9. In the expression $x^y = w$, the base is _____, the number is _____, and the logarithm is _____. 7_____ 8_____ 9_____

10. If the base is y, the number is a, and the logarithm is b, the expression may be written _____. 10_____

11. Show how to divide 256 by 32 by using powers of 2. 11_____

12. The characteristic is the _____ part of the logarithm. 12_____

13. The mantissa is the _____ part of the logarithm. 13._____

14. Logarithms to the base ten are called _____ logarithms. 14_____

15. $346 = 10^{?}$ 15_____

16. $5.49 = 10^{?}$ 16_____

17. $2.5 = 10^{?}$ 17_____

18. $8650 = 10^{?}$ 18_____

19. $(9.90)^3 = 10^{?}$ 19_____

20. $\sqrt[3]{2.97} = 10^{?}$ 20_____

21. $\dfrac{10^{9.1673}}{10^{7.6590}} = 10^{?}$ 21_____

Find the numbers which correspond to:

22. $10^{1.4133}$ 22_____

23. $10^{3.9096}$ 23_____

24. $10^{2.3945}$ 24_____

Use powers of ten to solve the following:

25. $3.27 \times 1.92$ 25_____

26. $8.00 \div 3.76$ 26_____

27. $(1.88)^4$ 27_____

28. $A = (42.5)^2 \times 3.14.$

A = 28_____

29. $K = \sqrt{s(s-a)(s-b)(s-c)}$

Find K when a = 14.6, b = 12. 7, and c = 15.8, and please do not ask if we forgot to tell you what s equals. 29_____

30. Kind K when $K = \frac{s^2}{4}\sqrt{3}$ and $s = 15.7$. 30_____

Test 2

Write in abbreviated logarithmic form: ($\text{Log}_b N = a$).

1. The log of sixteen to the base four is two. 1_____
2. The log of 128 to the base two is seven. 2_____
3. The log of z to the base x is y. 3_____
4. The log of 4 to the base 8 is 2/3. 4_____
5. The log of 45.6 to the base ten is 1.6590. 5_____

Write the following in the form: ($\text{Log}_b N = a$).

6. $5^4 = 625$ 6_____
7. $7^3 = 343$ 7_____
8. $5^2 = \frac{1}{25}$ 8_____
9. $6^0 = 1$ 9_____
10. $10^{2.8756} = 751$ 10_____
11. $x^y = w$ 11_____

Write in exponential form and solve for the unknown quantity.

| | Exponential Form | Value of the Unknown |
|---|---|---|
| 12-13. $\text{Log}_2 16 = x$ | 12_____ | 13_____ |
| 14-15. $\text{Log}_3 3 = y$ | 14_____ | 15_____ |
| 16-17. $\text{Log}_{10} 10{,}000 = n$ | 16_____ | 17_____ |
| 18-19. $\text{Log}_a a^2 = x$ | 18_____ | 19_____ |
| 20-21. $\text{Log } 8 = y$ | 20_____ | 21_____ |

(Following to be reworded or replaced)

| | | |
|---|---|---|
| 22-23. $3 \log_{10} 100 = n$ | 22_____ | 23_____ |
| 24-25. $5 \log_7 49 = x$ | 24_____ | 25_____ |
| 26-27. $\frac{1}{3} \log_3 27 = y$ | 26_____ | 27_____ |

Write in logarithmic form and solve for the unknown value.

28-29. $3^x = 27$ 28_____ 29_____

30-31. $5^x = 125$ 30_____ 31_____

32-33. $3^y = 243$ 32_____ 33_____

34-35. $2^y = 128$ 34_____ 35_____

36-37. $a^x = \sqrt{a}$ 36_____ 37_____

38-39. $a^y = 1$ 38_____ 39_____

Solve for the unknown.

40. $\text{Log}_2 2 = x$ 40_____

41. $\text{Log}_x 64 = 3$ 41_____

42. $\text{Log}_x 8 = 2$ 42_____

43. $\text{Log}_{16} 2 = x$ 43_____

44. $\text{Log}_4 N = \frac{1}{2}$ 44_____

45. $\text{Log}_2 32 = x$ 45_____

True or False ( T or F):

46. $\text{Log}_2 8 + \log_2 4 - \log_2 32 = 0$ 46_____

47. $\text{Log}_{10} 10 + 3 \log_{10} 100 - 2 \log_{10} 1000 = 0$ 47_____

48. $\text{Log}_9 27 + \log_{10} 1000 + \log_5 625 + \log_2 1 = 0$ 48_____

Find the value of x:

49. $\text{Log}_8 x = 1\frac{2}{3}$ 49_____

50. $x = \log_{\frac{1}{39}} \sqrt[5]{27}$ 50_____

Test 3

Use the tables on pages 472 and 473 of the text book.

Find the logarithms of:

1. 175 1_____
2. 4.1 2_____
3. 22.2 3_____
4. 2640 4_____
5. 0.68 5_____

Complete the following:

6. Log 2.67 = 6_____
7. Log 5,000,000 = 7_____
8. Log 61.6 = 8_____
9. Log 845 = 9_____
10. Log .06 = 10_____
11. Log (1.24 x 14.20) = 11_____
12. Log $\frac{947}{29.3}$ = 12_____
13. Log $\frac{396 \times 2470}{92.6}$ = 13_____
14. Log N = 0.3096; N = 14_____
15. Log A = 4.8241; A = 15_____
16. Log c = 2.9025; c = 16_____
17. Log r = 1.7427; r = 17_____
18. Log h = 7.9165-10; h = 18_____

Find the antilogarithms of the following logarithms:

19. 2.4728 19_____
20. $\bar{1}.9304$ 20_____

21. 3.4771 21_____

22. 0.9974 22_____

23. 8.7396 - 10 23_____

Interpolate when necessary to find the following. Do the computation in the margins or on the back of the preceding page. (24-28)

24. Log x = 2.3038; x = 24_____

25. Log n = 0.3749; n = 25_____

26. Log p = 3.9450; p = 26_____

27. Log A = 1.9761; A = 27_____

28. Log N = 7.9775 - 10; N = 28_____

Find the value to the fourth decimal place: (Find log only)

29. Log $36.5^2$ = 29_____

30. Log $\sqrt[4]{6}$ = 30_____

31. Log $\sqrt{36.5}$ = 31_____

32. Log $615^3$ = 32_____

33. Log $(1.02)^4$ = 33_____

34-38. Find the antilogarithms in the answers to 29-33 and record them for 34-38. (Interpolate) 34_____

35_____

36_____

37_____

38_____

39-43. Use the blank lines from 39-43 to complete the omissions in the set-up given below. 39_____

$T = 2\pi\sqrt{\frac{l}{g}}$ 40_____

41_____

Log T = log 2 ____ log $\pi$ ____ ____ 42_____

(log l ____ ____). 43_____

44. Solve for T in ex. 39, by logarithms, when l = 39.5 and g = 32.2. 44_____

45. If log x = 0.2230 and log y = 0.3721, log xy = _____. 45_____

46. In ex. 45, log $x^2y$ = 46_____

47. If $\log_{10} 42 = 1.6232$ and $\log_{10} 6 = 0.7782$, find $\log_{10} 7$ without tables. Show how you arrived at the answer. 47_____

48. Log N = 2 log 42.7 + 3 log 7.90 - log 7.62. What expression has been set up for logarithmic solution? 48_____

49. Solve exercise 48 by logs for N. 49_____

50. The side of an equilateral triangle is 29.7. Find the area by logs. 50._____

Test 4

1. Log 2635 = 1_____

2. Log 14.75 = 2_____

3. Log 0.9463 = 3_____

4. Log 247.9 = 4_____

5. Log 8.042 = 5_____

Refer to answers to the above as you do exercises 6-10.

6. Log (2635 x 14.75) = 6_____

7. Log $\frac{247.9}{8.042}$ = 7_____

8. Log $(14.75)^2$ = 8_____

9. Log $\sqrt[3]{8.042}$ 9_____

10. Log $\frac{\sqrt{2635}}{14.75}$ = 10_____

11. $\frac{1}{3}$ (8.9320-10) = 11_____

12. Log .0283 - log 283 = 12_____

13. Log .758 - log 247 = 13_____

14. Log 597 - log .0763 = 14_____

15. Log 2000 - log .525 15_____

16. N = 72.59; log N = 16_____

17. $\frac{1}{3}$ (11.8295-10) = 17_____

18. N = 300; log N = 18_____

19. X = 6249; Log X = 19_____

20. Y = .5295; log Y = 20_____

Refer to exercises 18 to 20 and solve by logs:

21. $n = N\sqrt{\frac{X\,N}{Y}}$ ; n = 21_____

22. By logs find $\sqrt[3]{100}$ 22_____

23. $x = (1.06)^{20}$; x = 23_____

24. $V = \frac{4}{3}\pi r^3$; Solve for r. 24_____

25. Fill in all vacancies in the set-up of the 25_____

solution to ex. 24. 26_____

Log r = ----- [(----- + -----) ----- 27_____

(----- ----- -----)] 28_____

29_____

30_____

31_____

32. Log .0299 = 32_____

33. Log 6.834 = 33_____

34. Log 12.95 = 34_____

35. Log 0.02496 = 35_____

Refer to exercises 32-35:

36. 6.834 (1.295) = 36_____

37. $\frac{0.02496}{0.0299}$ = 37_____

38. How many digits are there in $19^{20}$? 38_____

39. $\sqrt[3]{.0299}$ = (See ex. 32) 39_____

40-43. Log p = log 1700-9 log 1.04.

Log 1700 = 40_____

9 log 1.04 = 41_____

log p = 42_____

p = 43_____

44. Log 8.736 = 44_____

45. Log 3.142 = 45_____

46. The area of a sphere is $4\pi r^2$. Find the area of a sphere whose radius is 8.736 in. 46_____

47. The formula for the radius of the circle inscribed in a triangle is _____. 47_____

See exercise 47 for exercises 48-50.

48. Given a = 2420, b = 1570, and c = 1250; s = 48_____

49. Log r = 49_____

50. r = 50_____

## CHAPTER III

### Teaching the Unit on Logarithms

The teacher's log.-- A day by day account of the teaching of the unit on logarithms was kept in the form of brief notes written on the back of the pages of the teacher's copy of the pupil study guide. This log contained references to the work of particular pupils and notes concerning groups which were doing exceptionally good work or were running into difficulties. Sometimes pupil comments were jotted down but in such a way that the pupils were not conscious of it. Dates were recorded for pooling-of-experience programs and for tests. Correction of errors in typing which had slipped by in the first proof reading were made of spots which needed further explanation or less detailed directions. Indicated changes in order were also noted.

One comment jotted down at the close of the period when some of the slower pupils in the 11-3 class had worked for the first time alone on negative characteristics was, "Worked with enthusiasm after a restless start. Second group needs more help." Such brief comments were usually enough to enable the teacher to pick up the work the next day. In this particular instance she joined the members

of the second group of 11-3 to be sure that their difficulties were straightened out so that they could go along with confidence.

When there was a pooling-of-experience phase a pupil secretary took notes on the program and the teacher added brief notes so as to be able to report on the programs.

The chief value of the entries in the log was to enable the teacher to know which items in the study guide were worthy of being retained and which should be discarded or revised.

Introduction of the unit.-- The first lesson of the unit on logarithms was on Monday, March tenth. When the first few pupils entered the room, they noticed immediately the new display on the bulletin board at the back of the room and went to look it over. Other pupils joined them, while a few, who noticed the material on the table, asked if there was to be a test that period. Others, used to getting supplies the minute they entered, looked puzzled when the usual supplies and home work assignments were missing.

At the end of the five-minute traffic period many of the class had read the quotations from Napier's writings and had nearly completed the two-page article on logarithms which accompanied the portrait. The teacher had considered the possibility of this information influencing the scores

on the pre-test but had decided that the interest it would arouse would more than offset any premature instruction. Any pupil who remembered what he read so quickly would deserve the slight advantage it would give him in the pre-test.

Preliminary test.-- When the bell rang for the beginning of the period, the pupils who were still at the bulletin board or were looking at the exhibit in the bookcase walked toward their seats and joined in the informal conversation that was going on between the seated pupils and the teacher. The pupils were alert, curious, and seemed pleased to be starting logarithms.

When all were seated, they were given copies of the first part of the pre-test. Their facial expressions began to change. They varied from smug satisfaction on the face of one boy who had worked with a surveyor during the summer and calm acceptance on the part of the veterans, to apprehension and reproach on the faces of many of the other pupils. The teacher hastily called their attention to the foreword so that they would understand the purpose of the test. She also called to their attention the piles of pupil study-guides on the table and told them that when they had done as much as they could on the pre-test, they were to turn it in and take a copy of the study-guide. They could then spend the rest of the period discovering

how to use the study guide, in looking over reference books, or in completing their study of the bulletin board material.

The number of minutes spent on the test varied from ten to thirty-six, with twenty as an estimated average. The pupils were very serious about the work, and when they had turned in their test results, they were careful to move quietly so as not to interfere with those who were still working.

Pupil study guide.-- As each pupil turned in his test, he took a copy of the guide sheet as directed. Some pupils looked up the optional related activities immediately and were allowed to read them but were advised against starting any but the first one for at least a week.

The reference books had been numbered by the use of small gummed labels so that the pupils could obtain them more quickly than by referring to the reference list every time they wanted to find a book. R-5 and R-9 were in use for the rest of the period. By the end of the period some pupils had filled in the table on page one of the study guide and were working on page two. When they left the room, they asked if they could be allowed to continue the work at home, a strange contrast to their former method of slipping out quietly on the few occasions when the home assignment had not been given.

Second day of the unit.-- The second day, the early arrivals who evidently had read item 5 of the study guide, asked if they could rearrange the desks so as to work in groups. They were asked to wait until later in the period. However, they planned their groups and sent group leaders to obtain reference books so as to be prepared to start group work as soon as permission was given. The eight pupils in 11-1 and the ten in 11-3, who had participated in the geometry unit the previous year, gave impetus to the work.

When the second part of the pre-test was distributed, it was greeted with tolerant smiles. The teacher had decided that the psychological effect of obtaining an overall view of the unit from the study guides would outweigh the danger of influencing the results of the second part of the pre-test, particularly when the pupils did not know that such a test was to be given on the second day. Also, very little of Part II of the test could have been affected even though the pupils had studied many items of the study guide. However, some teachers might prefer to withhold the study guides until both sections of the pre-test were completed.

After only two minutes work eleven pupils in 11-1 turned in their papers, six worked for three minutes, four for four minutes, and three for five minutes. The

other class, 11-3, followed about the same pattern. After scanning the three pages of the test, and noticing that the work was evidently more advanced than what they had had time to do, they had returned the tests and had started to study. In the first class that left the two veterans and the young surveyor the only ones working on the test, so the teacher allowed the pupils who felt they did not need the first predetermined instruction point to work in groups provided they spoke only in whispers. The ten pupils who wanted help joined the teacher at the front board and in low voices worked on items two to five. When they felt they needed no more directed study they went to their seats to work from the study guide. At the end of thirty-five minutes, the last three tests had been handed in. Some pupils, before they left, had obtained change-of-room slips entitling them to report to the mathematics room during study periods so as to use the reserved books.

The third and fourth days.-- The third day the pupils had formed groups and were working on item 5 or more advanced items before the teacher entered the room. They asked for additional assignments in the text since there were not enough copies of the reference books and were told to add to the study guide the notation: T: 372: 10 exercises, 373: 1-4. They estimated that they would be ready for the first pooling-of-experience program on Friday and for the first test on Monday.

The teacher had hoped to convince them that they would be ready sooner, but when she discovered the difficulty of the exercises which many were choosing, she said nothing. Wednesday and Thursday she answered questions of individuals and of groups. Sometimes a group of pupils would send a representative to consult with the teacher, and on his return to the group they would ply him with questions. During class time the teacher was often requested to join a group to give approval or suggestions or to settle a doubt in some pupil's mind. By the end of the fourth period of work both classes were ready with contributions to the program on Friday.

First pooling of experience program.-- A pupil volunteered to act as secretary on Friday when the program was given. The pupils arranged their desks in group formation so as to consult among themselves when questions arose. The secretary's report showed these five exercises and their solutions for the first class:

1. $\dfrac{(9.9080)^5 \times (123.02)^2}{\sqrt[3]{158.67}} =$

2. $K = \sqrt{s\,(s-a)(s-b)(s-c)}$; find K when a = 12.62, b = 14.84, and c = 15.78.

3. 2.097 x 83 x 164 =

4. $L = \dfrac{t^2 g}{4\pi^2}$; find L when t = 2.860, g = 32.21, and $\pi = 3.142$.

$$5.\ N = \sqrt[4]{\frac{(32.41)^3\ (4.727)}{896.1\ \sqrt{1.769}}}$$

The exercises chosen by the second class were easier than the first and fifth chosen by 11-1, but otherwise were so much like them that they will not be repeated in this report.

There was no interpolation done as the guide sheet had directed the pupils to confine the work to three-digit numbers. Some pupils had inquired about exercises containing four or five-digit numbers and had been referred to items 20 and 24 of the study guide, but had also been told not to use interpolation in class discussions until later as it would confuse the greater number of pupils who had not gone that far. In the program, then, and in the first test, pupils rounded off four or five digit numbers to three-digit numbers. Therefore, the solutions of the first exercise in the program appeared as follows:

$$\frac{(9.91)^5 \times (123)^2}{\sqrt[3]{159}} = \frac{(10^{0.9961})^5 \times (10^{2.0899})^2}{} =$$

$$\frac{10^{4.9805} \times 10^{4.1798}}{10^{0.7338}} = \frac{10^{9.1603}}{10^{0.7338}} = 10^{8.4265} = 267,000,000$$

As it happened, the mantissa 4265 was in the table of mantissas. If it had not been, the pupils would have picked the number corresponding to the mantissa closest to 4265. Work on exponents had immediately preceded the work on

logarithms, so exponents to the base 10 presented no difficulties.

During the program the pupils took notes and asked questions informally. The pupil who had put an exercise on the board acted as chairman during the discussion of his work. When all the exercises had been completed, one of the pupils who had worked ahead said that the work could be done without so much writing. He spoke of the logarithmic form, but no more was said about it then so as not to confuse the pupils during the first test. They understood that the test was to be on the exponential form, but they knew that the next items in the study guide would introduce the logarithmic form.

The second week.-- The first test scheduled for the Monday following the first program was formulated by the teacher from items submitted by the pupils on Friday. This is Test 1 of chapter II. In some cases the questions were reworded by the teacher. There were thirty items in the test. The range of scores was 19 to 29, with a median score of 26.

Time was spent in class on Tuesday in discussing the test, not as a class recitation but as individual or group work so that no pupil was obliged to go over exercises which he had done correctly. Most of the time that period was spent on predetermined instruction point II, item 7.

Many pupils had studied as far as item 17, although they had not taken the tests. Such pupils were excused from the instruction periods, although some of them joined the instruction groups, as they explained, to check up on themselves. In some of the instruction periods, pupils who wished help gathered at the blackboard with the teacher; and the other pupils worked alone or in groups on the advanced work. At other times a few pupils worked at the sideboard or in seats at the back of the room on advanced work while the majority of the class and the teacher developed the new work.

One veteran was working on item 22, and the boy who had done summer work surveying was trying to reconcile his "rule-of-thumb" methods with the theoretical treatment, coming to the teacher with many questions. One veteran had scored high on Test 1, the second was the lowest in the class, and the young surveyor was third lowest. The slow veteran decided to work by himself and could not be persuaded to spend time studying the preliminary steps which he needed. He wanted to spend all his time on the diffcult work near the end of the unit and gave only cursory attention to the predetermined instruction pointsuntil his low marks in the tests shocked him into the realization that he had better get straightened out on the foundations.

Special difficulty.-- The most difficult part of item

7 was the solution of such exercises as $x = \frac{1}{3} \log_3 27$. Another year this kind of work will be omitted from item 7 and placed after item 28. Pupils will be firmly discouraged from trying these exercises at an early stage as experience showed that it slowed up the unit too much. If, as had been intended, pupils had solved easy cases of this type by inspection, the sequence of thought would not have been so interrupted. However, one bright-eyed group discovered two pages in R-7: 111 and 112, which the teacher had purposely left out of the study guide. This group was not happy until it had received help with such cases as $x = \log_3 \frac{1}{9} \sqrt[5]{27}$. The teacher knew that this was the prize question in one of the forthcoming tests but could not evade the issue raised by these pupils, who were some of those who were to take examinations for scholarships and for college entrance. The trouble was that less able pupils in other groups became curious as to what was going on, so Wednesday of the second week became a discussion and practice period on such work.

By the close of that period the teacher had convinced the classes that they were ready for the second test. The ill-fated discovery of R-7: 111-112 had made them fear that the test would be full of such exercises and they wanted more time for preparation. The teacher saw that the tentative time schedule would be affected adversely unless the slower pupils were discouraged from spending

more time on this type of exercise; so the teacher arbitrarily announced that the test was going to be given the next day. To reassure the worriers she told them that very few of the test items would be of that stage of difficulty.

It had been planned for pupils to take tests one to four individually whenever they felt they were ready for them, and this plan had been explained to them but the testing program did not work out that way. Pupils preferred to wait even though they were studying ahead and to take the tests as a class enterprise. One advantage to this was that no one liked to fall too far behind what the majority of the class was doing. The individual differences in such cases were expressed more in the difficulty of the exercises the pupils chose to solve than in the time of taking the tests.

The second test.-- On Thursday everyone took the second test, on the exponential and logarithmic forms of expression. There were fifty items in the test, and the scores ranged from 36 to 50 with a median of 48. There were four perfect scores due to the discovery of R-7: 111-112, from which the teacher, as explained before, had chosen the question which she had planned as the most challenging item. Rather than change the tests which had been prepared beforehand on the ditto machine, the question was given even though pupils had received help on it. The

surveyor's helper had improved so much that he was one of four pupils with scores of 49. The lowest score was that of the veteran who had chosen to work alone. The wording of items 22-27 in the test proved to be ambiguous and rather than discard the items, the teacher gave credit for correct responses to either of two possible interpretations and made a note in the log to reword the items before using the test another year.

The test was followed by a hasty oral check the next day to see how much of items 8 to 11 had been mastered and on reporting the results of the second test. The rest of the period was spent on group work in preparation for the twenty-minute quiz scheduled by the class for the following Monday.

The third week.-- The exercises chosen by the pupils for the quiz were from the text and the reference books and included the pendulum formula, volume of a cylinder, and compound interest, thus anticipating items 13, 16, and 35. In fact, much of the work in later items of the study guide appeared in this quiz in answer to some pupil's choice of an interesting exercise in the advanced work. Rather than rule out such exercises for an able pupil, the teacher referred him to the item in the study guide which would answer his difficulty. The pupil who had contributed an exercise to the quiz chose someone in a dif-

ferent group to solve the exercise on the board and designated other pupils to work on the same exercise at their desks. Thus, all questions in the quiz ere being solved simultaneously, and when the time to explain a solution arrived, all pupils who had worked on the exercise were encouraged to contribute to the discussion.

This quiz instead of yielding individual scores for the record book served as another pooling-of-experiences lesson and took the entire period instead of the twenty minutes which had been planned for it. Also, the quiz instead of serving only as a check of completed work, developed into a preview of later items for some pupils. One question raised was, "How do you subtract log 27.3 from log 1.25?" It was answered by a pupil who had been studying ahead of most of the class. The quiz also served as predetermined instruction point III as some of the exercises were so involved that the pupils saw the need of a careful set-up. At the end of the period pupils who had not reached items 13 to 16 planned to work on them for the next few days. Other pupils were free to work on more advanced items.

At this stage of the work the need of more bulletin board space became apparent. There was muc neat, accurate work which, if displayed, would be of value to other pupils. Some of the exercises of an aeronautical or

electrical source were especially interesting. It was at this time that the writer obtained large cardboard sheets and fastened them to the rear wall of the room and had pupils attach papers to them by Scotch tape. The members of 11-3 enjoyed checking the work of those pupils in 11-1 who were working on the same type of work as they were, so each class decided to submit its work to the members of the other class for the checking of the most difficult exercises they could find, and great was the glee when one class found an error in the work of the other.

Working with reference to a goal.-- On March twenty-fifth the teacher was to supervise the National Honor Society Scholastic Aptitude Test, and this necessitated her absence from the 11-1 class. A teacher from another department had been asked by the principal to "look in" on the mathematics class. "Look in" was all he had to do as the members of the class were so intent on setting up and solving difficult exercises that they paid no attention to the presence or absence of a teacher. The other teacher reported to the class teacher that he never had seen such a class. "They were working when I arrived and they never let up. What's the matter with them?" The mathematics teacher took the remark in the bantering way in which it was intended. However, her unexpressed opinion was that he had witnessed an illustration of what can be done when

pupils have adopted a goal which to them is worth while.

Negative characteristics.-- Although the pupils had been instructed to solve exercises involving numbers greater than one, the teacher had been receiving inquiries from advanced groups or individuals about logarithms of decimals. In these cases she had explained how to find the characteristic and had referred the pupils to items 17 and 19. By the end of the third week every pupil had reached this part of the work. The first three entries in the table had been typed incorrectly, probably through the attempt by the pupil typists to use the oblique rather than the horizontal line as the sign of the fraction, and this had slipped by in the first check of the stencils. Having the algebra students make the three necessary corrections on the study guide served as a good review of the definition of negative exponents. By the end of the third week for most pupils, and the first few days of the fourth week for all, item 21 had been finished.

Test 3.-- On April third test 3 was taken. The range was from 25 to 46 with the median score 38. The slower veteran had worked up the seventh from the lowest score, and the surveyor received the median score. No pupil had time to complete the fifty items of the test. The number of exercises attempted ranged from 34 to 49, with 44 the median number attempted. However, the test will not be

shortened for at least a year since the length of class periods has since been increased in the school. What will be done will be to see that bright pupils, who are apt to be impatient of practice such as was necessary here, are discouraged from attempting too many of the advance topics until they have spent more time on interpolating, with particular emphasis on "rounding off".

The facial expressions when the tests were returned, showed that on the whole the pupils were disappointed with their scores, and they responded well to directed practice then and there.

The rest of the fourth week was spent on items 22 to 25. The pooling-of-experience phase indicated in item 22 was omitted since much of this work had been discussed in the quiz which had followed item 12. The second interpolation practice in item 23 took little time. However, test 4 was not taken until April fourteenth as the slowest pupils kept requesting its postponement. Against her better judgment, the teacher allowed this postponement which meant that test 4 was given just before the final tests.

Last week of work.-- During the last week of actual work on the unit, the work was almost all done by conferences of individual pupils or small groups with the teacher, since pupils were working on different items from 25 to 38. The senior members of the class solved the numerical part of

their physics assignments in the mathematics class. The slowest members were disturbed when they found that the fifth week, a week longer than had been given in the tentative time, was definitely the final week of work.

The teacher had had qualms over letting the work extend so long. However, there had been no school on Friday of the fourth week; the greater part of one day and part of another had been given to pre-tests: four days had been devoted to tests one to four, and two more days would be given to the final tests. There were actually almost nine days taken from the twenty days as originally planned, which meant that there would have been only eleven days of work on the unit. Of the five weeks then, sixteen days were actually work days. Later, as will be explained in the evaluation of the unit, the teacher found that she saved time in the long run by not curtailing the time on logarithms.

To help the slowest pupils finish the unit without too much dissatisfaction, the teacher excused them from much of the work in items 26, 29, 31, 32, 33, and 34. Some of these pupils as noted in chapter I could not have been expected, on the basis of personal records and previous grades in mathematics, to accomplish much except with the easiest parts of the unit. They worked hard, however, and would willingly have attempted more except that time did not permit it. The teacher saw to it that items 35 and 36

on compound interest were studied by all pupils. The unit was formally completed on Wednesday and Thursday of the week of April fourteenth when parts I and II of the final test were given. It will be recalled that the final test was the same as the pre-test. The last activity was the administration of a questionnaire to discover the pupil's reactions to the unit.

The questionnaire will be found in the next chapter on the evaluation of the unit.

## CHAPTER IV

## EVALUATION OF THE UNIT ON LOGARITHMS

### Statistical Treatment of Test Results

The pre-test.-- Table 10 of Chapter I and Table 22 of Appendix B furnished the data for the statistical treatment of all test results. Table 11 shows the wide range in the scores in the pre-test. The highest score was 78, the lowest was 0, and the arithmetic mean was 24.73.

Table 11. Arithmetic Mean and Standard Deviation of the Total Scores in the Pre-test of Fifty-two College-Mathematics Pupils in Logarithms in a Grouped Frequency Distribution

Class Interval = 5

| Limits | Midpoint | Frequency f | Deviations d | Frequency Deviations fd | $fd^2$ |
|---|---|---|---|---|---|
| 76-80 | 78 | 1 | +12 | +12 | 144 |
| 71-75 | 73 | 0 | +11 | 0 | 0 |
| 66-70 | 68 | 0 | +10 | 0 | 0 |
| 61-65 | 63 | 0 | + 9 | 0 | 0 |
| 56-60 | 58 | 0 | + 8 | 0 | 0 |
| 51-55 | 53 | 4 | + 7 | +28 | 196 |
| 46-50 | 48 | 4 | + 6 | +24 | 144 |
| 41-45 | 43 | 3 | + 5 | +15 | 75 |
| 36-40 | 38 | 3 | + 4 | +12 | 48 |
| 31-35 | 33 | 2 | + 3 | + 6 | 18 |
| 26-30 | 28 | 3 | + 2 | + 6 | 12 |
| 21-25 | 23 | 3 | + 1 | + 3 | 3 |
| 16-20 | 18 | 10 | 0 | 0 | 0 |

Table 11. (Concluded)

Class Interval = 5

| Limits | Midpoint | Frequency f | Deviations d | Frequency Deviations fd | $fd^2$ |
|---|---|---|---|---|---|
| 11-15 | 13 | 6 | -1 | - 6 | 6 |
| 6-10 | 8 | 11 | -2 | -22 | 44 |
| 1- 5 | 3 | 0 | -3 | 0 | 0 |
| - 0 | - | 2 | -4 | - 8 | 32 |
| | | N=52 | | $\Sigma fd=70$ | $\Sigma fd^2=722$ |

Arithmetic Mean

Assumed mean = 18

$$\frac{\Sigma fd}{N} = \frac{70}{42} = 1.346$$

$$\frac{\Sigma fd}{N}(c.i.) = 5(1.346)$$

$$= 6.730$$

$$M = 18 + 6.730$$

$$= 24.730$$

Standard Deviation

$$S.D. = c.\sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2}$$

$$\frac{\Sigma fd}{N} = \frac{722}{52} = 13.88$$

$$\left(\frac{\Sigma fd}{N}\right)^2 = (1.346)^2 = 1.812$$

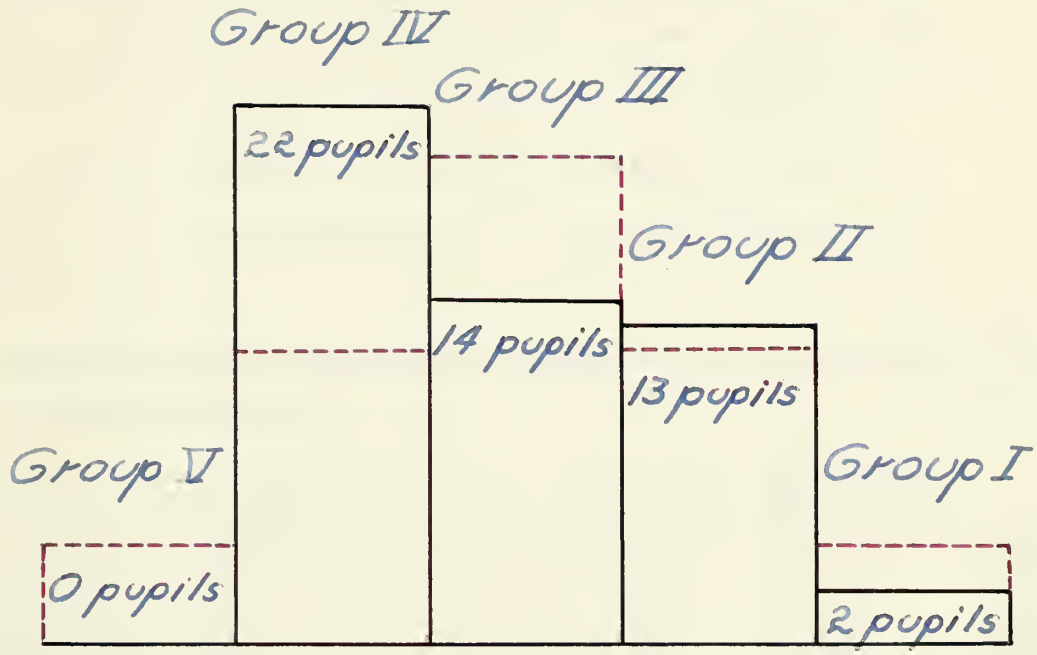
$$S.D. = 5\sqrt{13.88 - 1.812}$$

$$= 5\sqrt{12.07}$$

$$= 5(3.47) = 17.35 = 17.4$$

Figure 4 depicts the relative scores grouped about the mean and also the actual distribution and standard distribution of scores on the pre-test. The score of 78 fell outside the range of Group I. These results, when compared with the corresponding table and figure for the final test (Table 13 and Figure 6) show significant gains in Groups I, II, and III.

The gains in the scores in the final examination over the scores in the pre-test are treated in Table 12




Actual distribution of pupils on pre-test in logarithms with normal distribution in red.

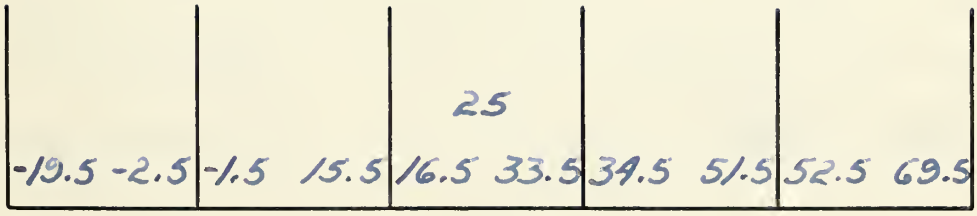



Figure 4. Relative growth scale - Pre-test on logarithms.

and in Figure 5.

Table 12. Arithmetic Mean and Standard Deviation of Gains in the Final Examination over the Pre-test in the Total Scores of Fifty-two College-Mathematics Pupils in Logarithms in a Grouped Frequency Distribution.

Class Interval = 5

| Limits | Midpoint | Frequency f | Deviations d | Frequency Deviations fd | $fd^2$ |
|---|---|---|---|---|---|
| 91-95 | 93 | 1 | +6 | + 6 | 36 |
| 86-90 | 88 | 4 | +5 | +20 | 100 |
| 81-85 | 83 | 5 | +4 | +20 | 80 |
| 76-80 | 78 | 2 | +3 | + 6 | 18 |
| 71-75 | 73 | 5 | +2 | +10 | 20 |
| 66-70 | 68 | 7 | +1 | + 7 | 7 |
| 61-65 | 63 | 9 | 0 | 0 | 0 |
| 56-60 | 58 | 2 | -1 | - 2 | 2 |
| 51-55 | 53 | 3 | -2 | - 6 | 12 |
| 46-50 | 48 | 2 | -3 | - 6 | 18 |
| 41-45 | 43 | 2 | -4 | - 8 | 32 |
| 36-40 | 38 | 5 | -5 | -25 | 125 |
| 31-35 | 33 | 2 | -6 | -12 | 72 |
| 26-30 | 28 | 3 | -7 | -21 | 147 |
| 21-25 | 23 | 1 | -8 | - 8 | 64 |
| Totals | | | | -19 | 733 |

Arithmetic Mean

Assumed mean is 63

Standard Deviation

$$S.D. = s\sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2}$$

Table 12. (Concluded)

$$\frac{\Sigma fd}{N} = \frac{-19}{52} = -.365 = c$$

$$\frac{\Sigma fd}{N}(c.i.) = 5(-.365) = -1.825$$

$$M = \text{Assumed mean} + (-1.825)$$
$$= 63 - 1.825$$
$$= 61.175$$

$$= 5\sqrt{\frac{733}{52} = (-.365)^2}$$
$$= 5\sqrt{14.0961 - .1332}$$
$$= 5\sqrt{13.9629}$$
$$= 5\ (3.74)$$
$$= 18.70$$

The range of gains extended from 95 to 25; the arithmetic mean was 61.18 and the standard deviation was 18.70. This does not represent a gain in computational ability only, but an increase in concepts and understandings also. Figure 5 indicates that the actual distribution of gains approximates very closely the normal expectancy. The difference of one or two cases is not especially significant when the number of pupils is fifty-two and results are rounded off to the nearest unit. Comments on the scores of individual pupils will be made later in the chapter.

The final test.-- Total scores on the final test are treated in Table 13 and in Figure 6.

Table 13. Arithmetic Mean and Standard Deviation of Total Scores of Fifty-two College Mathematics Pupils on Logarithm Examination in a Grouped Frequency Distribution.

Class Interval = 5

| Limits | Midpoint | Frequency f | Deviations d | Frequency Deviations fd | $fd^2$ |
|---|---|---|---|---|---|
| 111-115 | 113 | 1 | +5 | 5 | 25 |
| 106-110 | 108 | 1 | +4 | 4 | 16 |
| 101-105 | 103 | 3 | +3 | 9 | 27 |
| 96-100 | 98 | 5 | +2 | 10 | 20 |
| 91- 95 | 93 | 8 | +1 | 8 | 8 |
| 86- 90 | 88 | 11 | 0 | 0 | 0 |
| 81- 85 | 83 | 9 | -1 | -9 | 9 |
| 76- 80 | 78 | 5 | -2 | -10 | 20 |
| 71- 75 | 73 | 6 | -3 | -18 | 54 |
| 66- 70 | 68 | 2 | -4 | - 8 | 32 |
| 61- 65 | 63 | 0 | -5 | 0 | 0 |
| 56- 60 | 58 | 0 | -6 | 0 | 0 |
| 51- 55 | 53 | 1 | -7 | - 7 | 49 |
| | | | | +36 | |
| | | | | -32 | |
| | | N=52 | | $\Sigma fd =$ -16 | $\Sigma fd = 260$ |

**Arithmetic Mean**

Assumed mean = 88

$$\frac{\Sigma fd}{N} = \frac{-16}{42} = -.308 = c$$

$$\frac{\Sigma fd}{N}(c.i.) = 5(-.308) = -1.540$$

M = assumed mean + (-1.540)
= 88 - 1.540
= 86.46

(68% of scores fall between 97.5 and 75.41
34% - 14% - 2 %)

**Standard Deviation**

$$S.D. = s\sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2}$$

$$= 5\sqrt{5-}$$

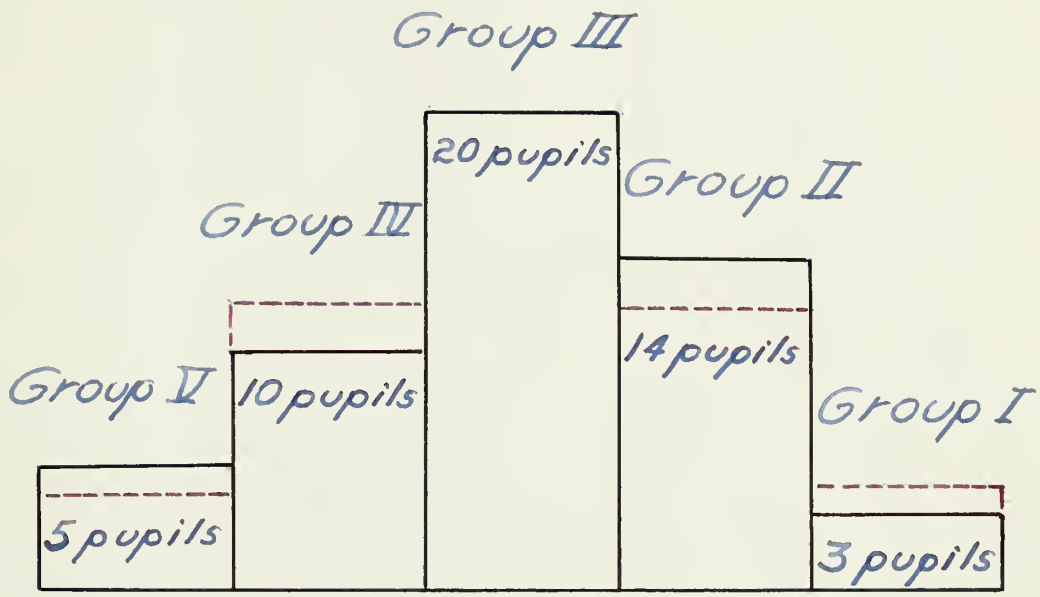
$$\frac{\Sigma fd}{N} = \frac{260}{52} = 5.0000$$

$$\left(\frac{\Sigma fd}{N}\right)^2 = c^2 = (-.308)^2 = .0949$$

$$S.D. = 5\sqrt{5.000 - .0949}$$

$$= 5(2.21)$$

$$= 11.05$$




Actual distribution of pupil gains on final test on logarithms with normal distribution in red.

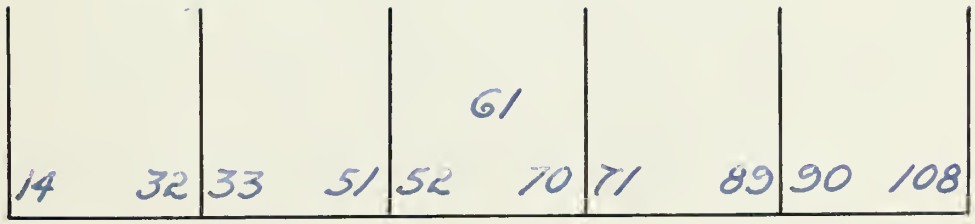
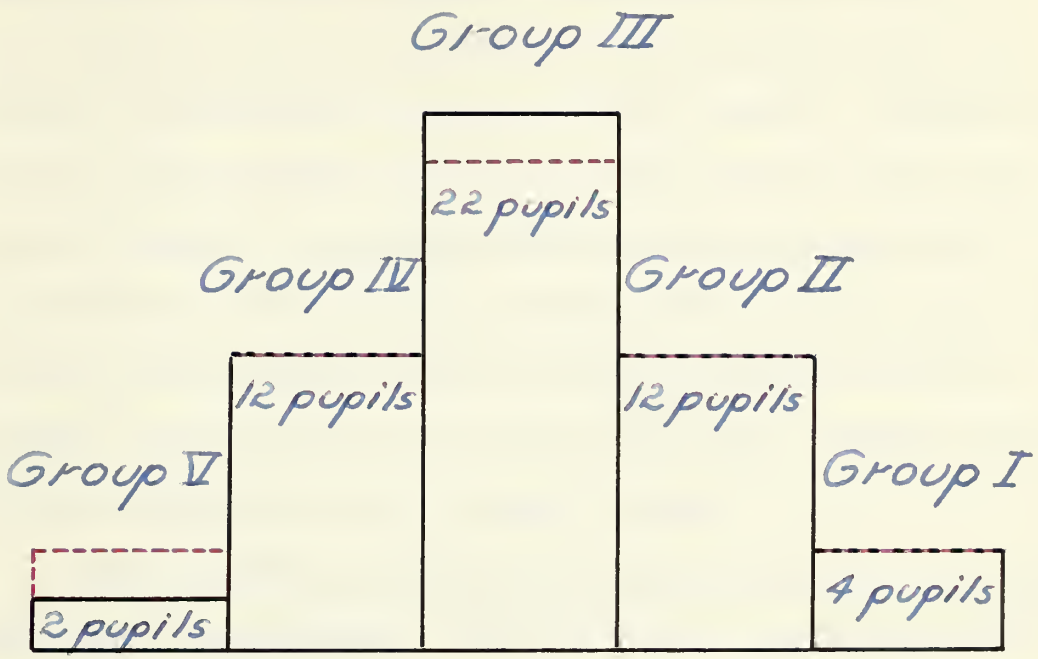



Figure 5. Relative growth scale for gains on logarithm test.




Actual distribution of pupil growth with normal distribution in red.

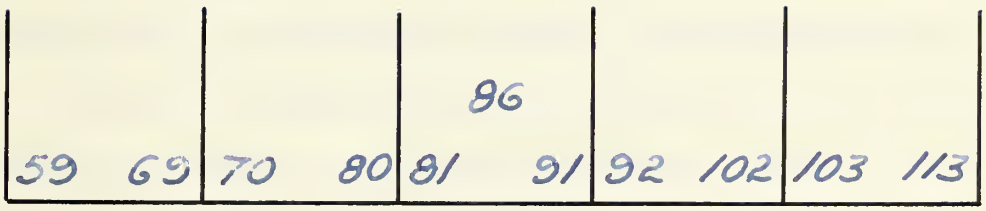



Figure 6. Relative growth scale for total scores on logarithm test.

The total scores extended from 111 to 51; the arithmetic mean was 86.46; the standard deviation was 11.05. In this instance the actual distribution agreed with the normal distribution in Groups I, II, and IV. There were fewer cases in Group V in the actual distribution than in the normal. As before, these interpretations must be considered as having been derived from a small number of cases in the sample, but the writer considers them of enough significance to use the tests with another group before making any major changes in them.

Coefficient of correlation for Part I and Part II of the final examination.-- Part I of the examination was composed of ninety test items designed to test concepts and understandings. It did not require the use of tables, and any necessary computation was of an elementary nature. Part II contained thirty items which required pupils to use tables, to set up exercises in logarithmic form, and to perform computations of a more exacting nature. It tested actual performance as well as understandings.

In view of the above, the coefficient of correlation between the pupils' positions in Part I and Part II of the test could serve as an indication as to whether this particular unit developed understandings, concepts, and the ability to use them correctly in actual situations. Actually, the correlation was not so simple, as other

factors, such as intelligence, attendance, previous training in mathematics, and personality traits were involved. However, assuming the foregoing factors to be constant for Part I and for Part II of the test, and using the Spearman formula, the coefficient of correlation was found to be 0.69. This may be considered a moderately good correlation. The writer realizes that the presence of the duplicate scores affects the accuracy of the result. However, as a rough indication of correlation and with all the qualifications mentioned before, the correlation 0.69 tends to show that in this case, the unit method developed both understandings and concepts and the ability to use them correctly. Neither purpose was developed independently of the other.

Accomplishment of Individual Pupils

Pre-test scores.-- The high score of 78 in the pre-test was the score of one of the veterans who was taking the course as a refresher course before he entered college. Before the completion of the school year, he was accepted for Northeastern University. He was the third highest pupil in the final test on logarithms.

The next two highest scores on the pre-test, 53 and 52, respectively, were achieved by an eleventh-grade boy and another veteran. Oddly enough, these two pupils had final scores of 92 and 91, respectively. They were in

different divisions and so did not work together on the unit in school, nor did they have any interests in common out of school, yet their scores closely resembled one another. These boys were in the seventeenth and eighteenth positions in the final test.

The younger boy was quick acting, curious, and impatient of sustained effort. His intelligence quotient was 131. He was interested in all branches of athletics sponsored by the school, the American Legion, and his church. His advance knowledge of logarithms was explained by his association with older boys who coached some of the athletic teams. At odd moments they worked on school assignments, and this boy's curiousity led him to listen and to ask questions.

The other boy was a veteran who worked slowly and methodically. His intelligence quotient was 115, and previous to entering the service his marks in mathematics had been C, L, and L. He had repeated first year algebra with L for a mark the second time. He had studied mathematics while in the Marine Corps and had taught the subject in a public high school in Honolulu. His ambition was to earn a high-school diploma so as to enter college to study engineering.

There were only two pupils with total scores of zero in the pre-test. The first was a girl with intelligence

quotient of 129. All of her interests were along literary lines. Her final score was 51, the second lowest final score, but the thirty-eighth highest in the gains made.

The other zero score in the pre-test belonged to a girl whose intelligence quotient was 117. She was a professional skater and also worked for a caterer. In spite of poor attendance, her final score was 89 points, the second highest in the number of points gained.

Final scores.-- The highest total score of 111, with a gain of 63 points, belonged to a senior girl. This girl had attended five different schools in another state before she entered the tenth grade. Her intelligence quotient was 131, and she was active in six important extra-curricular activities. These included the position of secretary of the student council, an editorship of the year-book, and membership in the National Honor Society. Her out-of-school activities included sailing, swimming, instruction in diving and in driving a car, music lessons, and church activities. She was one of the girls who was accepted for Radcliffe College.

The second highest pupil in the final test was a girl who had worked in the same group as the girl described above. She was an eleventh grade pupil with an intelligence quotient of 135. She also had an extensive list of extra-curricular and out-of-school activities. She was

interested in no particular college at that time but later made application to Duke University.

The other pupil in Group I of the actual distribution was the veteran who had the highest score in the pre-test and who was described there. The presence of veterans and seniors in the eleventh grade class set a high standard of work for the other pupils.

The boy who had worked during the summer with the civil engineer, and who was referred to in the account of the teaching of the unit, had a score of 28 in the pre-test and of 94 in the final test. He was one of four pupils in the eighth highest place in the final test and so was in Group II in the actual distribution of pupil growth.

The veteran who had worked alone had a score of 51 in the pre-test and of 80 in the final test which placed him in Group IV. His intelligence quotient was 91, and with the exception of a C in eighth grade mathematics, all of his previous grades had been L's. As a ninth grade pupil he had a grade placement of 7.9 in reading comprehension, 7.6 in arithmetic computation, and 8.2 in language. During the war he had served as gunner's mate in the Atlantic theater of operations. By the end of the school year he had decided to become a traveling salesman, rather than an engineer.

The two pupils in Group V of the final test were a

girl and a boy. The girl was one of those who had zero in the pre-test and was described in the pre-test section of this report.

The boy might have been expected to be in Group IV or V. His intelligence quotient was 101, reading vocabulary 7.6, language 6.6. His previous marks in mathematics had been B, B, C, and B, but the last two marks had been in shop courses in mathematics. He had changed to the college-preparatory program in the eleventh grade and had been allowed to elect college mathematics. His ambition was to be a "mechinical engineer". He had a fine disposition and was a hard worker, but the mathematics teacher did not see eye-to-eye with the guidance department in his case.

Other pupils described in Chapter I.-- The second pupil who was accepted for Radcliffe College was in Group III in the final test on logarithms. Her score was 90 points. She had an intelligence quotient of 128 and previous marks in mathematics of B, A, B, B. She had taken no mathematics in the tenth grade and geometry in the eleventh. Her chief interests were literary and dramatic, and all her extra-curricular activities were in these fields. Her ambition was to enter the diplomatic service or to become an actress.

The girl who entered Smith College the next year

wanted to train for the foreign service. She had attended schools at Denver, Colorado; San Francisco, California; New York City; Montreal and Toronto, Canada; and in Paris and London. Her father was a construction engineer and the family had accompanied him as he traveled in his work. The girl's intelligence quotient was 123 and her previous marks had been C, B, A, and B. She had had no mathematics the previous year and had pretty well forgotten her algebra. She was one of the youngest pupils in the class, fifteen years and seven months, but her graciousness and poise made her appear older. Her final score was 85, one point below the median.

The girl who later entered the Chandler School to study to be a medical secretary was in Group II with a final score of 98 points.

Guidance function of the unit.-- The success of the pupils in being accepted by the colleges of their choice might be considered an indication that their eleventh-grade classmates who had comparable or better scores, and who ranked well in their other subjects and in school activities would probably be successful in gaining admittance to college.

In this connection it may be noted that of the forty-two eleventh grade pupils of the total fifty-two who studied the unit, twenty-five elected solid geometry and

trigonometry for their senior year. With very few exceptions, pupils in Group IV and Group V did not elect these subject. It would seem that the unit on logarithms, as was the case with the unit on similar triangles, was important from the standpoint of guidance. The pupils were brought face to face with their own abilities or limitations in a situation where they could see that they had been given every chance to succeed.

## Pupil Opinion of the Unit

The questionnaire:-- At the close of the unit a questionnaire was given to the pupils. There had been no advance notice that this was to be done, and the responses were written without discussion, so it is believed that pupils gave unbiased replies. Seven pupils were absent the day the questionnaires were given.

The questions were as follows:

1. Do you prefer the unit method or the traditional method of learning mathematics?
2. Do you work harder with either method? If so, with which one?
3. Which method do you think is more fun?
4. Do you think you learn more with one method or the other? If so, with which one?
5. Which method would you prefer for the new work on trigonometry?

6. What suggestions would you give for improving either method?
7. What do you like best about the unit method?
8. What do you consider its worst feature?

Pupil responses to the questionnaire.-- Pupil responses to the items in the questionnaire are given in Table 14.

Table 14. Replies to the Questionnaire Given at the Completion of the Unit on Logarithms.

| Question in Condensed Form | Replies | | |
|---|---|---|---|
| | Either Method | Unit Method | Traditional Method |
| 1. Preference | --- | 41 | 4 |
| 2. Work harder | 14 | 19 | 12 |
| 3. More fun | 2 | 43 | 0 |
| 4. Learn more | 12 | 31 | 2 |
| 5. Choice for trigonometry | 3 | 37 | 5 |
| | Replies in Order of Frequency | | |
| 6. Suggestions for improving unit method. | 1. More reference books or faster circulation of reference books. | | |
| | 2. More class discussion or intergroup discussions. | | |
| | 3. Less class discussion | | |

Table 14. (Continued)

| Question in Condensed Form | Replies in Order of Frequency |
|---|---|
| | 4. Definite home-work each day (pupil with low ability). |
| | 5. More predetermined instruction points (pupil who had had no mathematics the preceding year). |
| | 6. More time given in class to reports on optional related activities. |
| 7. Best feature of unit method. | 1. Can work at your own speed. |
| | 2. Can ask more questions and get more individual help. |
| | 3. Can ask questions of members of your group and not feel that the class would consider them trivial. |
| | 4. Talk over problems as soon as they arise. |
| | 5. Get help on questions that seem too trivial to ask the teacher. |
| | 6. Can plan homework to suit one's own schedule. |
| | 7. Time for more actual practice in school time. |
| | 8. Chance to use different books. |
| | 9. Know exactly what to study for tests. |
| | 10. Have definite notes to rely on. |

Table 14. (Concluded)

| Question in Condensed Form | Replies in Order of Frequency |
|---|---|
| 8. Worst feature of unit method. | 1. Not enough reference books. |
| | 2. Tests too long. |
| | 3. Temptation for lazy people to shirk work. |
| | 4. Temptation for social talk in groups. |
| | 5. Too much time in class instruction on work which we know (bright pupils). |
| | 6. Not enough class discussion (slower pupils and some lazy pupils). |
| | 7. Study guide bulky. |

Of the forty-five pupils who answered the questionnaire four to one expressed a preference for the unit method. Fourteen pupils felt that they worked hard no matter which method was used. Of the others, the vote was nineteen to twelve in regard to working harder with the unit method.

The two votes for no difference as to the amount of fun or enjoyment might indicate that the two pupils enjoyed either method or again it might signify that they saw no fun in either method. However, the vote of forty-three pupils for the unit method as providing more fun is very significant.

Twelve pupils saw no difference in method in the amount learned, but thirty-one to two thought that they learned more by the unit method than by the traditional method.

The choice of method for the next topic was thirty-seven to five for the unit method with only three impartial as to method.

Thus, in the responses to everyone of the first five questions the unit method received by far the most pupil votes.

Of the pupils' suggestions for improving the unit, the one most frequently given was the need for more reference books. In response to this, the school department approved an order for three copies each of five reference books, and the books were received the next September.

Some pupils wanted more class discussion and some wanted less. The brightest pupils were the ones who suggested less class discussion.

The comments on the best feature of the unit method, as given in Table 14, agree closely with the opinions expressed by the pupils who studiedthe unit on similar triangles. Since these responses were given a very detailed treatment in Part I, they will not be discussed again.

The comments on the worst features were closely related to replies to question 6 of the questionnaire. The second

comment to this item, as given in Table 14 was "Tests too long". This was justified in view of the forty-one minute period. The next two comments, "Temptation for lazy people to shirk work", and "Temptation for social talk in groups", showed that the pupils recognized that much of the responsibility for their success was placed in their hands and that it was expected that they would react in the right way. However, the teacher should be alert to this situation and, as indicated in the discussion of the teaching of the unit, be ready to guide any group which gives indication of shirking or of wasting time on too much sociability.

Quotations from pupils' replies.-- The following quotations are from the pupils' replies to the questionnaire:

"I think that you learn more with the unit method because you can use a wide variety of reference books and also check your work with other members of your group."

"The unit method does not force you to do certain work for the next day, but I find that I do more work if it isn't required at a certain time."

"The unit method offers a possibility to get out of homework, but the interested will study without looking for chances to dodge work, and the uninterested will take advantage of any system."

"I do about the same amount of work in each method, but I understand more with the unit method. The thing I like best about the unit method is your chance to concentrate on one thing until you know it."

"The unit method gives you a chance to work with your classmates and break down the stiff problems together."

"Traditional method - more teaching. Unit method - more asking."

The pupil who wrote the last opinion went right to the heart of the matter. He noticed that the teacher activity of the traditional method was replacedby pupil activity in the unit method.

Final comments.-- The comments given at the conclusion of Part I in regard to the unit on similar triangles apply equally well to the unit on logarithms. There was the same enthusiasm on the part of the pupils, the same interest in planning their work, and the same concentration in the way in which they carried out their plans. Here, as before, there was a great deal of hard work accomplished in an atmosphere of well-being and good will.

Although at first glance it might appear that the unit on logarithms took more time than some teachers would care to spend on it, a survey of the work done would disclose that what they would classify as review work was actually a part of the unit. This would include the

formulas of plane geometry, of physics, and the informal treatment of the formulas of three-dimensional geometry. Moreover, much time was saved later in the course in studying geometric progressions and trignometry since no time had to be spent in a review of logarithms. Pupils used logarithms with understanding and facility whenever they were necessary in later work.

A distinctive feature of the unit on logarithms as the teacher looks back upon it, was the wealth of applications which were available to the pupils. Logarithms were learned in a natural setting, not as an isolated tool to be used sometime in the future. Since the pupils progressed at their own rates, the ablest pupils were not kept on practice work which was needed by slower pupils. They were free to follow the pupil study guide and to consult their textbooks and reference books for mathematical experiences which involved logarithms. Many geometric concepts were kept alive in their minds since formulas formerly solved by arithmetic could be done more easily with logarithms. No one textbook could have supplied the electrical, aeronautical, and other functional situations in which many pupils were interested. Moreover, the traditional classroom method would not have been suitable for the exploration and experimentation conducted by pupils in regard to the various logarithmic tables.

The final conclusions expressed in the unit on similar triangles apply also to the unit on logarithms. It can be said again here that the unit method set the stage and gave direction to happy, industrious pupils who worked avidly in the direction of a goal which they had adopted because they had deemed it to be meaningul to them and necessary to the accomplishment of their objectives in life.

# APPENDIX A

Table 15. Intelligence Quotients and Previous Marks in Mathematics of Pupils in Group I of the Unit on Similar Triangles

| Pupil | Intelligence Quotient | Previous Marks in Mathematics Grade 7 | 8 | 9 |
|---|---|---|---|---|
| (1) | (2) | (3) | (4) | (5) |
| 1 | 165 | A | A | A |
| 2 | 145 | B | A | A |
| 3 | 139 | C | L | C |
| 4 | 139 | B | C | L |
| 5 | 135 | A | A | A |
| 6 | 131 | C | B | C |
| 7 | 130 | A | B | C |
| 8 | 130 | A | A | C |
| 9 | 127 | A | A | A |
| 10 | 125 | B | C | L |
| 11 | 124 | B | B | L |
| 12 | 124 | C | C | L |
| 13 | 122 | C | B | C |
| 14 | 120 | C | B | C |
| 15 | 120 | C | C | L |
| 16 | 119 | --- | --- | --- |
| 17 | 115 | B | A | B |
| 18 | 115 | C | B | C |
| 19 | 115 | A | B | B |
| 20 | 114 | --- | A | C |
| 21 | 114 | L | C | B |
| 22 | 114 | L | B | L |
| 23 | 114 | B | B | B |
| 24 | 111 | B | B | C |
| 25 | 110 | B | B | C |
| 26 | Not available | | | |
| Median | 122 | B | B | C |

Table 16. Intelligence Quotients and Previous Marks in Mathematics of Pupils in Group II of of the Unit on Similar Triangles

| Pupil | Intelligence Quotient | Previous Marks in Mathematics Grade 7 | 8 | 9 |
|---|---|---|---|---|
| (1) | (2) | (3) | (4) | (5) |
| 1 | 146 | B | B | C |
| 2 | 142 | A | A | B |
| 3 | 139 | A | A | A |
| 4 | 137 | B | B | C |
| 5 | 133 | B | B | B |
| 6 | 132 | C | B | C |
| 7 | 131 | B | L | C |
| 8 | 129 | C | A | C |
| 9 | 127 | C | C | C |
| 10 | 127 | A | B | L |
| 11 | 124 | C | B | C |
| 12 | 123 | C | B | C |
| 13 | 123 | B | B | B |
| 14 | 122 | B | B | L |
| 15 | 120 | A | A | A |
| 16 | 117 | B | B | C |
| 17 | 117 | B | C | C |
| 18 | 117 | C | C | C |
| 19 | 113 | L | C | D,C |
| 20 | 112 | C | B | B |
| 21 | 111 | B | B | C |
| 22 | 109 | C | C | C |
| 23 | 108 | A | B | A |
| 24 | 106 | C | C | B |
| 25 | 100 | C | C | C |
| 26 | 98 | A | A | B |
| 27 | 97 | L | L | L |
| Median | 122 | B | B | C |

Table 17. Similar Triangle Examination

Section A

Groups I and II Combined

| Number of Test Items Correct | Pupil Frequencies | Number of Test Items Correct | Pupil Frequencies |
|---|---|---|---|
| 72 | 1 | 53 | 2 |
| 71 | 1 | 52 | 4 |
| 70 | 2 | 51 | 2 |
| 69 | 1 | 50 | 1 |
| 68 | 1 | 49 | 2 |
| 64 | 2 | 47 | 4 |
| 62 | 1 | 46 | 2 |
| 61 | 5 | 45 | 1 |
| 59 | 4 | 43 | 2 |
| 58 | 2 | 39 | 1 |
| 56 | 1 | 31 | 1 |
| 55 | 1 | 25 | 1 |
| 54 | 1 | | |

| | | | |
|---|---|---|---|
| High Score | 72 | Total Frequencies | 53 |
| Low Score | 25 | Range | 47 |
| | | Median Score | 54 |

Table 18. Similar Triangle Examination

Section B

Groups I and II Combined

| Number of Test Items Correct | Pupil Frequencies | Number of Test Items Correct | Pupil Frequencies |
|---|---|---|---|
| 17 | 2 | 9 | 10 |
| 16 | 1 | 8 | 8 |
| 15 | 1 | 7 | 4 |
| 14 | 2 | 6 | 3 |
| 13 | 2 | 5 | 4 |
| 11 | 3 | 4 | 1 |
| 10 | 6 | 3 | 1 |
| | | Total frequencies | 53 |
| High Score | 17 | Range | 14 |
| Low Score | 3 | Median Score | 9 |

Table 19. Analysis of Test Items

of Similar Triangle Examination

Section A

| Test Item | Frequencies of Correct Responses | Incorrect Responses | Omission of Item | Omission because of lack of time |
|---|---|---|---|---|
| (1) | (2) | (3) | (4) | (5) |
| 1 | 49 | 4 | | |
| 2 | 50 | 3 | | |
| 3 | 52 | 1 | | |
| 4 | 53 | 0 | | |
| 5 | 50 | 3 | | |
| 6 | 53 | 0 | | |
| 7 | 46 | 7 | | |
| 8 | 53 | 0 | | |
| 9 | 39 | 14 | | |
| 10 | 38 | 15 | | |
| 11 | 47 | 6 | | |
| 12 | 32 | 21 | | |
| 13 | 48 | 5 | | |
| 14 | 49 | 4 | | |
| 15 | 49 | 4 | | |
| 16 | 37 | 16 | | |
| 17 | 22 | 31 | | |
| 18 | 26 | 27 | | |
| 19 | 44 | 9 | | |
| 20 | 43 | 10 | | |
| 21 | 46 | 7 | | |
| 22 | 18 | 35 | | |
| 23 | 42 | 11 | | |
| 24 | 20 | 32 | 1 | |
| 25 | 52 | 1 | | |
| 26 | 37 | 16 | | |
| 27 | 29 | 24 | | |
| 28 | 46 | 6 | 1 | |
| 29 | 40 | 13 | | |
| 30 | 49 | 4 | | |

Table 19. (Continued)

| Test Item | Frequencies of Correct Responses | Incorrect Responses | Omission of Item | Omission because of lack of time |
|---|---|---|---|---|
| (1) | (2) | (3) | (4) | (5) |
| 31 | 32 | 21 | 0 | |
| 32 | 28 | 23 | 2 | |
| 33 | 41 | 12 | 0 | |
| 34 | 24 | 29 | 0 | |
| 35 | 26 | 27 | 0 | |
| 36 | 30 | 22 | 0 | 1 |
| 37 | 22 | 16 | 4 | 1 |
| 38 | 24 | 25 | 3 | 1 |
| 39 | 47 | 2 | 3 | 1 |
| 40 | 34 | 18 | 0 | 1 |
| 41 | 44 | 8 | 0 | 1 |
| 42 | 33 | 19 | 0 | 1 |
| 43 | 46 | 6 | 0 | 1 |
| 44 | 36 | 16 | 0 | 1 |
| 45 | 44 | 8 | 0 | 1 |
| 46 | 43 | 9 | 0 | 1 |
| 47 | 46 | 6 | 0 | 1 |
| 48 | 27 | 25 | 0 | 1 |
| 49 | 43 | 9 | 0 | 1 |
| 50 | 50 | 1 | 1 | 1 |
| 51 | 28 | 23 | 0 | 2 |
| 52 | 40 | 11 | 0 | 2 |
| 53 | 28 | 23 | 0 | 2 |
| 54 | 24 | 27 | 0 | 2 |
| 55 | 27 | 24 | 0 | 2 |
| 56 | 35 | 16 | 0 | 2 |
| 57 | 33 | 18 | 0 | 2 |
| 58 | 18 | 33 | 0 | 2 |
| 59 | 33 | 18 | 0 | 2 |
| 60 | 37 | 12 | 2 | 2 |
| 61 | 31 | 18 | 2 | 2 |
| 62 | 23 | 26 | 2 | 2 |
| 63 | 24 | 25 | 1 | 3 |
| 64 | 46 | 3 | 1 | 3 |
| 65 | 30 | 19 | 1 | 3 |

Table 19 (Concluded)

| Test Item | Frequencies of Correct Responses | Incorrect Responses | Omission of Item | Omission because of lack of time |
|---|---|---|---|---|
| (1) | (2) | (3) | (4) | (5) |
| 66 | 27 | 16 | 6 | 4 |
| 67 | 15 | 28 | 6 | 4 |
| 68 | 23 | 26 | 0 | 4 |
| 69 | 16 | 32 | 0 | 5 |
| 70 | 35 | 9 | 3 | 6 |
| 71 | 25 | 16 | 0 | 12 |
| 72 | 32 | 9 | 0 | 12 |
| 73 | 16 | 25 | 0 | 12 |
| 74 | 18 | 17 | 4 | 14 |
| 75 | 14 | 19 | 5 | 15 |
| 76 | 21 | 12 | 4 | 16 |
| 77 | 17 | 14 | 5 | 17 |
| 78 | 32 | 1 | 3 | 17 |
| 79 | 12 | 11 | 2 | 28 |
| 80 | 20 | 4 | 1 | 28 |
| 81 | 19 | 4 | 1 | 29 |
| 82 | 17 | 7 | 0 | 29 |
| 83 | 16 | 8 | 0 | 29 |
| 84 | 13 | 9 | 0 | 31 |
| 85 | 15 | 5 | 0 | 33 |

Table 20. Analysis of Test Items of

Similar Triangle Examination

Section B

| Test Item | Frequencies of Correct Responses | Incorrect Responses | Omission of Item | Omission because of lack of time |
|---|---|---|---|---|
| (1) | (2) | (3) | (4) | (5) |
| 1 | 47 | 6 | | |
| 2 | 49 | 3 | 1 | |
| 3 | 49 | 4 | | |
| 4 | 48 | 5 | | |
| 5 | 32 | 7 | 14 | |
| 6 | 26 | 18 | 9 | |
| 7 | 47 | 5 | 1 | |
| 8 | 42 | 10 | 1 | |
| 9 | 31 | 21 | 1 | |
| 10 | 15 | 21 | 17 | |
| 11 | 12 | 18 | 23 | |
| 12 | 27 | 13 | 12 | 1 |
| 13 | 13 | 16 | 23 | 1 |
| 14 | 8 | 16 | 28 | 1 |
| 15 | 4 | 27 | 20 | 2 |
| 16 | 22 | 20 | 5 | 6 |
| 17 | 6 | 24 | 11 | 12 |
| 18 | 7 | 19 | 11 | 16 |
| 19 | 24 | 10 | 2 | 17 |
| 20 | 1 | 9 | 4 | 39 |
| 21 | 0 | 6 | 0 | 47 |

Table 21. Raw Scores of Final Examination

on Similar Triangles

Total Scores

Section A and Section B

| Score | Frequency | Score | Frequency |
|---|---|---|---|
| 86 | 1 | 59 | 2 |
| 85 | 1 | 58 | 3 |
| 83 | 1 | 56 | 1 |
| 81 | 1 | 55 | 2 |
| 80 | 3 | 54 | 3 |
| 78 | 1 | 53 | 1 |
| 77 | 1 | 51 | 1 |
| 75 | 1 | 49 | 1 |
| 73 | 1 | 46 | 1 |
| 72 | 2 | 44 | 2 |
| 71 | 1 | 39 | 1 |
| 70 | 2 | 36 | 1 |
| 69 | 2 | 34 | 1 |
| 68 | 1 | | |
| 67 | 3 | | |
| 65 | 3 | | |
| 63 | 3 | | |
| 62 | 3 | | |
| 61 | 1 | | |
| 60 | 1 | | |

N = 53

Highest Score 86

Lowest Score 34

Range 52

Comments on Final Examination

Section A

Test items.-- An analysis of the results of Section A of the final examination on similar triangles indicated that some of the test items should be reworded. It would improve item 25 to add the words, "At the same time".

Item 38 was confusing to the best pupils. Some of them omitted it. Many of the poorer pupils had it right, which indicated that they might have guessed at it. It was meant to be a difficult question but failed in its purpose and will be replaced by 2r (10-r) = 36.

Item 39 contained a typographical error in the spelling of circle, but it did not interfere with the understanding of the item. (Table 18).

The wording of item 46 should include the expression "without changing the value of x." There was an error in typing item 41. "C" should have been "c".

Item 58 was meant to be a trick question to see if the pupils noted the importance of the word "respectively", and will not be reworded, although thirty-three pupils gave the wrong response. (Table 18).

In item 61 the change from "inches" to "units" was intentional, but many of the best pupils interpreted it as an error in typing and replied accordingly. This item the pupils regarded as unfair, so it will be changed for

the next test. They regarded item 58 as a fair item although many of them had been caught by it.

In item 66 it would have been clearer had the angles and segments been identified by letters.

Item 67 would be a good item to be proved in another examination or to be substituted in place of items 71 to 78, or 79 to 85. Pupils could be required to mark it true or false. An assigned number of points of credit could be given for the proof of the truth or falsity of the item.

In the pupils' mimeographed copy of the test, more than half of page four had been left vacant because items 71 to 78 and the figure to which they referred had to be arranged so that no pages would be turned until item 78 had been completed. Some pupils did not realize that the test was continued on the next page, so the directions, "Turn to the next page", will be typed on such pages in the future.

Some pupils neglected to cross out the false items although they circled the correct ones. Such responses were counted as omitted. Some scores registered carelessness in following directions rather than lack of knowledge of geometry.

Length of the examination.-- Pupils found the test too long for a forty-one minute period. (Table 18). Since a committee of the faculty had been appointed to

study the advisability of lengthening periods, the teacher decided to await the outcome of the committee report before deleting the test. The second year after the teaching of the unit, the period was lengthened to forty-nine minutes, which was more satisfactory for all phases of the unit method.

Comments on Final Examination

Section B

Length of examination.-- This examination also proved to be too long for the time allotted to it. (Table 19). There had not been an opportunity to have the questions mimeographed as the commercial department was busy with other work, and since it had been very cooperative in preparing the six-page copies of Section A, the geometry teacher did not feel free to request more work done for her classes. Therefore, the examination was written on the blackboard and pupil time was wasted in the mechanics of the examination. Since then a ditto machine has been purchased for the use of teacher, so examinations may be prepared without overburdening the commercial department. Also, the length of the school period was extended the second year after the unit was taught, so better results may be expected the next time the examination is used.

Item 5.-- The response of Group II to item 5 indicated that the wording was ambiguous. The next time the examination is used, the item will read, "The numerical value of k is _____."

Shadow measurement.-- Items 7 and 8, on shadow measurement were answered correctly by 47 and 42 pupils respectively. Item 26 in Form A which was almost identical with item 8 in Form B was answered correctly by 37 pupils.

It may be that the other pupils knew the principle involved but were handicapped by a language difficulty. They applied the principle correctly but did not recognize it when stated abstractly.

Partial answers.-- Items 10 and 11 were incorrect on some papers because pupils recorded a partial answer as the final answer.

Item 15.-- The poor response to item 15 was a surprise to the teacher as it was a comparatively easy question of a type commonly found in textbooks, standardized tests, and examinations of the College Entrance Examination Board. It called only for the solution of a right triangle. The pupils may have been thrown off the track and lost time in trying to use similarity. This type question had not appeared in any practical setting in the unit, which may further explain the trouble it caused in the examination. The teacher decided that the next time the unit was used this type question would receive more prominence chiefly as insurance against examinations which would be prepared outside of the school.

Gear problem.-- Group II did better than Group I with items 17 and 18. It was in Group II that the broken gear had been brought in, and items 17 and 18 were solved by means of the same theorem as had been used for the gear problem. Group I had not taken as direct interest in this

problem as had Group II and this was reflected in the test results. In section A of the test item 30 had tested knowledge of this theorem. Every pupil in Group I and all but four in Group II had answered it correctly. Here was an instance which illustrated that knowledge of the theory did not guarantee the ability to apply it correctly in practice. Those who had taken a keen interest in its first application were the ones who recognized its application to a similar, but not identical, situation.

Last three items.-- Item 19 was easy and probably would have been answered correctly by more pupils had it been placed earlier in the test. Seventeen pupils did not have time to do it. (Table 19). However, since the test items followed the sequence of the delimitation of the unit, the position of this item will not be changed. Items 20 and 21 will not be changed either, because, with a longer school period indicated, these items will help to supply enough work and difficult enough work to challenge the best pupils. It might be advisable to add to the test one or two exercises of the degree of difficulty usually reserved for eleventh-grade pupils.

# APPENDIX B

Key to Test 1

1. John Napier (Napier accepted)
2. Henry Briggs (Briggs accepted)
3. False
4. 16 or 1614
5. addition
6. subtraction
7. x
8. w
9. y
10. $y^b = a$
11. $\frac{2^8}{2^5} = 2^3 - 8$
12. integral
13. fractional
14. common
15. 2.5391
16. 0.7396
17. 0.3979
18. 3.9370
19. 2.9868
20. 0.1576
21. 1.5083
22. 25.9
23. 8120
24. 248
25. 6.28
26. 2.13
27. 12.5
28. 5670
29. 88.3
30. 107

Note: Interpolation was not used at this stage of the work.

Key to Logarithm Examination

Part I

The actual key was arranged so that the responses fitted beside the test items, but for convenience in typing the responses have been arranged in successive columns.

| Item | Response | I | R | I | R | I | R | I | R | I | R |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | + | 16 | + | 31 | + | 46 | - | 61 | + | 76 | + |
| 2 | - | 17 | - | 32 | + | 47 | + | 62 | + | 77 | - |
| 3 | - | 18 | + | 33 | + | 48 | + | 63 | - | 78 | + |
| 4 | + | 19 | + | 34 | - | 49 | - | 64 | + | 79 | + |
| 5 | - | 20 | + | 35 | + | 50 | - | 65 | - | 80 | - |
| 6 | - | 21 | - | 36 | + | 51 | - | 66 | + | 81 | - |
| 7 | + | 22 | - | 37 | + | 52 | - | 67 | + | 82 | + |
| 8 | + | 23 | - | 38 | - | 53 | - | 68 | + | 83 | - |
| 9 | + | 24 | - | 39 | + | 54 | - | 69 | + | 84 | + |
| 10 | - | 25 | + | 40 | - | 55 | + | 70 | - | 85 | - |
| 11 | - | 26 | - | 41 | + | 56 | + | 71 | - | 86 | + |
| 12 | + | 27 | - | 42 | + | 57 | - | 72 | + | 87 | - |
| 13 | + | 28 | - | 43 | + | 58 | - | 73 | + | 88 | - |
| 14 | - | 29 | + | 44 | + | 59 | - | 74 | + | 89 | + |
| 15 | - | 30 | + | 45 | - | 60 | - | 75 | + | 90 | + |

Key to Logarithm Examination

Part II

| Item | Response | Item | Response |
|---|---|---|---|
| 91. | 1.5145 | 106. | .3842 |
| 92. | 7.4771-10 | 107. | 751.8 |
| 93. | 941.0 | 108. | $\frac{1}{2}$ |
| 94. | 0.4580 | 109. | 2 |
| 95. | 0.1691 | 110. | - |
| 96. | 0.5175 | 111. | log 4827 |
| 97. | 5.590 | 112. | 5.7516 |
| 98. | 4.5915 | 113. | 564400 |
| 99. | + log 2.78 - log 1.26 | 114. | Neatness |
| 100. | 9.1250-10 | 115. | 985.2 |
| 101. | 9.1590-10 | 116. | 2.9583 |
| 102. | 8.9538-10 | 117. | 908.4 |
| 103. | 40 | 118. | Neatness |
| 104. | 2.448 | 119. | $2917 |
| 105. | 8.7537-10 | 120. | $10^{\sqrt{2}}$ (Satisfactory at this stage) |

Key to Test 2

1. $\text{Log}_4 16 = 2$
2. $\text{Log}_2 128 = 7$
3. $\text{Log}_x z = y$
4. $\text{Log}_8 4 = \frac{2}{3}$
5. $\text{Log}_{10} 45.6 = 1.6590$ or $\text{Log } 45.6 = 1.6590$
6. $\text{Log}_5 625 = 4$
7. $\text{Log}_7 343 = 3$
8. $\text{Log}_5 \frac{1}{25} = -2$
9. $\text{Log}_b 1 = 0$
10. $\text{Log}_{10} 751 = 2.8756$
11. $\text{Log}_x w = y$
12. $2^x = 16$
13. 4
14. $3^y = 3$
15. 1
16. $10^n = 10{,}000$
17. 4
18. $a^x = a^2$
19. 2
20. $2^y = 8$
21. 3

22 to 27. To be reworded or replaced.

28. $\text{Log}_3 27 = x$

29. 3

30. $\text{Log}_5 125 = x$

31. 3

32. $\text{Log}_3 243 = y$

33. 5

34. $\text{Log}_2 128 = y$

35. 7

36. $\text{Log}_a \sqrt{a} = x$

37. $\frac{1}{2}$

38. $\text{Log}_a 1 = y$

39. 0

40. 1

41. 4

42. $2\sqrt{2}$

43. $\frac{1}{4}$

44. 2

45. 5

46. T

47. F

48. F

49. 32

50. $-\frac{7}{5}$

# Key to Test 3

| | | | |
|---|---|---|---|
| 1. | 2.2430 | 24. | 201.3 |
| 2. | 0.6128 | 25. | 2.371 |
| 3. | 1.3464 | 26. | 8810 |
| 4. | 3.4216 | 27. | 94.65 |
| 5. | 9.8325-10 | 28. | .009495 |
| 6. | 0.4265 | 29. | 3.1246 |
| 7. | 6.6990 | 30. | 0.1946 |
| 8. | 1.7896 | 31. | 0.7812 |
| 9. | 2.9269 | 32. | 8.3667 |
| 10. | 8.7782-10 | 33. | 0.0344 |
| 11. | 1.2457 | 34. | 1332 |
| 12. | 1.5094 | 35. | 1.565 |
| 13. | 4.0238 | 36. | 6.042 |
| 14. | 2.04 | 37. | 232600000 |
| 15. | 66700 | 38. | 1.082 |
| 16. | 799 | 39. | + |
| 17. | 55.3 | 40. | + |
| 18. | .00825 | 41. | $\frac{1}{2}$ |
| 19. | 297 | 42. | - |
| 20. | .852 | 43. | log g |
| 21. | 3000 | 44. | 6.96 |
| 22. | 9.94 | 45. | 0.5951 |
| 23. | .0549 | 46. | 0.8181 |

47. Log 42 - log 6 = log 7; 0.8450

48. $\frac{42.7^2\ (7.90)^3}{7.62}$

49. 117,900

50. 382

Key to Test 4

| | | | |
|---|---|---|---|
| 1. | 3.4208 | 24. | $\sqrt[3]{\frac{3v}{4\pi}}$ |
| 2. | 1.1688 | 25. | $\frac{1}{3}$ |
| 3. | 9.9760-10 | 26. | log 3 |
| 4. | 2.3943 | 27. | log v |
| 5. | 0.9054 | 28. | - |
| 6. | 4.5896 | 29. | log 4 |
| 7. | 1.4889 | 30. | + |
| 8. | 2.3376 | 31. | log $\pi$ |
| 9. | 0.3018 | 32. | 8.4757-10 |
| 10. | 0.5416 | 33. | 0.8347 |
| 11. | 9.6440-10 | 34. | 1.1122 |
| 12. | 6.0000-10 | 35. | 8.3972-10 |
| 13. | 7.4870-10 | 36. | 8.85 |
| 14. | 3.8935 | 37. | .8346 |
| 15. | 3.5808 | 38. | 26 |
| 16. | 1.8608 | 39. | .3104 |
| 17. | 0.6098 | 40. | 3.2304 |
| 18. | 2.4771 | 41. | 0.1530 |
| 19. | 3.7958 | 42. | 3.0774 |
| 20. | 9.7239-10 | 43. | 1195 |
| 21. | 564,400 | 44. | 0.9413 |
| 22. | 4.642 | 45. | 0.4972 |
| 23. | 3.206 | 46. | 959.2 |

47. $r = \frac{K}{2}$

48. 2620

49. 2.5203

50. 331.2

Table 22. Distribution of Raw Scores in Logarithm Test

| Score | Frequency | Score | Frequency |
|---|---|---|---|
| 111 | 1 | 87 | 3 |
| 110 | 1 | 86 | 1 |
| 104 | 1 | 85 | 2 |
| 103 | 1 | 84 | 2 |
| 101 | 1 | 83 | 3 |
| 100 | 2 | 81 | 2 |
| 98 | 1 | 80 | 1 |
| 96 | 2 | 78 | 1 |
| 94 | 4 | 77 | 2 |
| 93 | 2 | 76 | 1 |
| 92 | 1 | 74 | 1 |
| 91 | 1 | 73 | 1 |
| 90 | 1 | 72 | 3 |
| 89 | 22 | 71 | 1 |
| 88 | 4 | 70 | 1 |
| | | 68 | 1 |
| | | 51 | 1 |

N = 52

| | |
|---|---|
| High Score | 111 |
| Low Score | 51 |
| Range | 61 |

Illustrations

Bulletin board display.-- The first illustration entitled "Bulletin Board Display to Introduce the Unit on Logarithms" shows the material which the pupils saw for the first time when they entered the room for the introductory lesson to the second unit. This material was described fully in the second chapter of Part II.

Illustration of pupils at work.-- In the second illustration, "Pupils Working in Groups on the Unit on Logarithms," part of the first class may be seen. An indication of the physical equipment of the room is also part of the picture. The bookcase contained reference books, one of which was being consulted by the girl standing at the left. On the top shelf of the bookcase were logarithmic tables and models of pyramids and cones, the latter to accompany the solution of exercises which concerned these figures.

The two pupils standing together were arranging a bulletin board display of work which they had completed and which they were submitting for criticism by members of other groups.

The six girls seated beoyond the standing pupils worked sometimes as one group, but the day the picture was taken they were working in two separate groups. The two boys in the foreground were using the table of logarithms in the textbook.

Copies of the study guides may be noted in the picture. One may be seen under the opened book of the boy in the right foreground, and another in the corner of the desk of the boy at the extreme right of the second row. Two girls in the second row were discussing work in the study guide and one girl was making notations in pencil in her copy.

By observing the different books on the desks of the pupils, one may see that the groups were working at various stages of the unit and had reached different phases of achievement. The girl who was examining the reference material in the bookcase was starting on a new application of the work. Those at the bulletin board were ready for criticism of completed work. The other pupils were at various stages between these two.

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